5U LEVEL II (10)

DNA 4904F

AD A090217

STATISTICAL-DIMENSIONAL ANALYSIS: AN APPLICATION TO THE ASSESSMENT OF CRATER CONFIGURATION

Boeing Company
P.O. Box 3999
Seattle, Washington 98124

31 January 1979

Final Report for Period 19 June 1978—31 January 1979

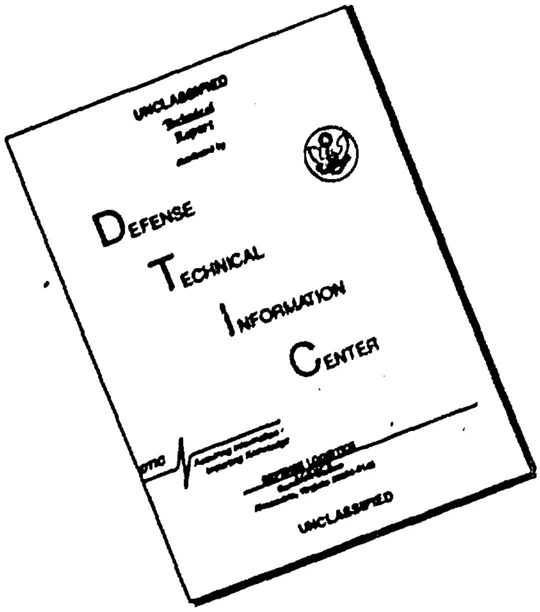
CONTRACT No. DNA 001-78-C-0326

APPROVED FOR PUBLIC RELEASE;
DISTRIBUTION UNLIMITED.

THIS WORK SPONSORED BY THE DEFENSE NUCLEAR AGENCY UNDER RDT&E RMSS CODE B344078464 Y99QAXSA00216 H2590D.

Prepared for
Director
DEFENSE NUCLEAR AGENCY
Washington, D. C. 20305

DISCLAIMER NOTICE



THIS DOCUMENT IS BEST QUALITY AVAILABLE. THE COPY FURNISHED TO DTIC CONTAINED A SIGNIFICANT NUMBER OF PAGES WHICH DO NOT REPRODUCE LEGIBLY.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

17

1. REPORT NUMBER 18 DNA 4904F	2. GOVT ACCESSION NO. AD-A090217	3. RECIPIENT'S CATALOG NUMBER 9
4. TITLE (and Subtitle) 6 STATISTICAL-DIMENSIONAL ANALYSIS: AN APPLICATION TO THE ASSESSMENT OF CRATER CONFIGURATION.		5. TYPE OF REPORT & PERIOD COVERED Final Report, [redacted] 19 Jun 78—31 Jan 79, 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) 10 R. M. Schmidt K. A. Holsapple L. D. Fisher		8. CONTRACT OR GRANT NUMBER(s) 15 DNA 001-78-C-0326 new
9. PERFORMING ORGANIZATION NAME AND ADDRESS Boeing Company P. O. Box 3999 Seattle, Washington 98124		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 16 Subtask Y99QAXSA002-16
11. CONTROLLING OFFICE NAME AND ADDRESS Director Defense Nuclear Agency Washington, D. C. 20305		11 12. REPORT DATE 31 Janu[redacted]979 13. NUMBER OF PAGES 94
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) 12 96		15. SECURITY CLASS (of this report) UNCLASSIFIED 15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

This work sponsored by the Defense Nuclear Agency under RDT&E RMSS Code B344078464 Y99QAXSA00216 H2590D.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Apparent Crater Volume	Gravity Scaling
Cratering	Material Properties
Crater Scaling	Scaling Laws
Dimensional Analysis	Statistical Analysis

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Definition of crater related environments is influenced by a large number of variables potentially governing the phenomena. The large body of data is derived from many events of different character. A method to exploit this heterogeneous body for prediction is presented. Methods of dimensional analysis are used to reduce the number of independent variables through the introduction of nondimensional π-groups. More importantly, this allows a common basis for comparing data from grossly dissimilar data bases. These

20. ABSTRACT (Continued)

include field events having different soil, stratigraphy and moisture content, and different explosive type, size and configuration, as well as laboratory data in accelerated reference frames. Successive combination of nondimensional variables is then accomplished using a step-wise regression analysis to generate scaling laws which provide relationships among the variables. Because all the various data are compared on a common basis, the statistics resulting from a large base provide higher confidence in predictions as the quantitive uncertainties are less than for each data base considered separately. Correlation of apparent crater volume was used as a pilot model to demonstrate the technique.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

SECTION 1
INTRODUCTION

Various regression schemes have been used with limited success to analyze empirical explosive cratering data and so define scaling relationships. The objective of this study is to improve the precision of these scaling laws using combined statistical and dimensional analysis techniques that result in functional forms relating a simplified set of generalized variables.

The approach used here is to first develop a set of independent nondimensional parameters from the independent physical variables of the phenomenon through dimensional analysis. For example, dimensional analysis of recent centrifuge experimental data on cratering lead to the identification of a charge size-parameter (Schmidt, 1977). A statistical regression analysis is then performed to determine functional relationships among these parameters.

The common obstacle defeating previous attempts to unify this type of data was an inability to correlate the large number of independent variables necessary to describe the phenomenon. In this study, dimensional analysis provides a method to systematically combine groups of variables, allowing a common comparison of grossly different types of experiments, leading to mathematical models for the statistical regression analysis. The combined approach is a new technique incorporating recent scaling developments that allow simpler functional forms based upon fewer generalized variables. This hybrid methodology leads to improved confidence in prediction and quantification of associated uncertainties.

One of the few comprehensive statistical approaches to correlating different soil media is an earlier work by Dillon (1971). He tried to combine data for various types of materials by using a large number of parameters to relate soil properties. It is not totally clear how the regression was run, but it is clear that his model was far too complex for any useful prediction. Dillon states that every effort was made to keep the number of parameters to 40 or less (page 43). These are too many parameters for a problem of this type and one would have little confidence in any extrapolation. It is not surprising that his multiple correlation coefficient values are quite large when fitting a

model with 40 or so parameters. However, the values given appear even higher than the quality of the data fit on the plots suggests. This along with the fact that two of the values exceed 100, a theoretical impossiblity, indicates either severe numerical problems or a programming error. Dillon's work is weak statistically in the area of combining the different soil materials and very little reliability can be placed upon that portion of the research.

The other limitation imposed by Dillon's approach was his ad hoc choice of using a 5/16 yield scaling exponent for all linear dimensions. In the present study, scaling laws are determined directly by the statistical regression on the appropriate non-dimensional π-groups. These latter quantities serve as a set of generalized variables and require no ad hoc scaling exponent; but rather, as is shown in Section 3, theoretical analysis alone can assign appropriate scaling exponents for various size regimes (see Table 3 and Figs. 7, 8 and 9).

The study described here incorporates a particular material strength theory developed under a concurrent DNA centrifuge experimental program (Schmidt and Holsapple, 1979). This feature of the analysis significantly improved correlation, greatly reducing the scatter for cratering events in different media.

SECTION 2
BACKGROUND ON STATISTICAL MODEL FITTING

2-1 THE "TRUE" RELATIONSHIP AND EXPERIMENTAL VARIABILITY

In Physics, other than atomic physics, it is assumed that if all relevant information and appropriate physical laws were known, experimental findings could be predicted without error. Thus, this must also be true in relating explosions to craters. In point of fact, not all the relevant relationships are known. The data are not only affected by measurement error but also by inherent variablility in the explosive and in the cratering medium. Further, it is presently beyond human capability, to measure every relevant quantity. The "true" relationship between explosions and cratering dimensions is consequently masked by variability in the data.

2-2 ESTIMATING THE UNDERLYING "TRUE" RELATIONSHIP

Because there is variability, investigators cannot hope to discover the appropriate relationship and to fit all of the data points with complete precision. Since a completely accurate prediction cannot occur, a model is fitted to the data that explains much of the variation in the data. Statistical model fitting is the science and art of finding a mathematical model to fit data with random variability. An example of such data is the cratering data used in this report.

In general, the mathematical model used to fit data will depend upon a finite number of unknown constants called coefficients or parameters. The first step is to estimate the unknown coefficients from the data. The second step is to determine whether the model does give a reasonable fit to the data. The model can then be used to predict observed values before additional data are collected. The accuracy of the prediction depends upon the correctness of both the mathematical model and the statistical model for variability. Checking the fit of the model and the appropriateness of assumptions of random variability allows quantification of the variability in a future prediction.

2-3 LEAST-SQUARES CRITERIA FOR FITTING DATA

Investigators want to fit the "best" possible curve to the data. To find a best fit there is a need for some criterion with which to measure the goodness of the fit, such as the method of least squares. First proposed by Gauss, it is mathematically tractable in many situations and can be easily calculated. If the variation about the true curve is distributed according to a probability distribution (called the Gaussian, normal, or bell-shaped curve), the method picks the curve which makes the probability of the observed data as large as possible. This is also called the method of maximum likelihood.

To examine this method, consider Fig. 1 where some response (dependent variable y) is observed whose value may fluctuate with the value of the independent variable x. Consider now a family of proposed curves to model the data y; the curves give y as some function of x and the parameter θ. Three parametric curves are drawn corresponding to values of $\theta = 1$, $\theta = 2$, and $\theta = 3$. The data points being fitted are given by the four triangles. Suppose the curve corresponding to $\theta = 1$ were to represent these data. Corresponding to an x value of x_1, the appropriate y value on the curve is given by $\hat{y}_1$. Similarly corresponding to x_2, the y value on the curve is given by the value $\hat{y}_2$. The difference between the observed value y_1 and the value for $\hat{y}_1$ predicted by x_1 is the quantity y_1 minus $\hat{y}_1$. The principle of least squares directs us to examine the sum of the squares of the deviations of the data points minus their predicted values. In this case that sum is given by eq. 1.

$$\text{Sum of Squares} = (y_1 - \hat{y}_1)^2 + (y_2 - \hat{y}_2)^2 + (y_3 - \hat{y}_3)^2 + (y_4 - \hat{y}_4)^2 \qquad (1)$$

This sum of squares can be computed for each of the three curves given in Fig. 1. Of course, as changes between the three curves are made, the value $\hat{y}_1$ predicted for y_1 will change. Of the three, the curve corresponding to $\theta = 2$ would have the minimum value for the sum of squares. Thus according to the least-squares principle, the curve with $\theta = 2$ is chosen in preference to $\theta = 1$ or $\theta = 3$. In general, the dependence of the family of curves upon the parameters may be quite complex. Although the numerical problem of finding the curve that minimizes the sum of squares may be difficult, the method is straight

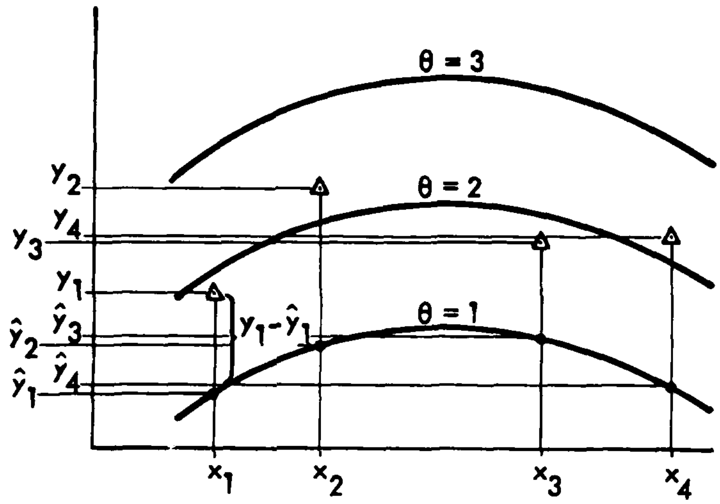


Fig. 1 Three possible curves, $y = f(x,\theta)$, for a least squares fit to four data points

forward. The difference between the observed y and the predicted $\hat{y}$ for the best fit is called the residual. The differences are called residuals because this term measures the amount of variability left over and not explained by the model used to fit the data, i.e. residual variability.

2-4 POTENTIAL PROBLEMS WITH THE METHOD OF LEAST-SQUARES FITTING

Though the method of least-squares fitting has proved useful, it is not a solution for all data analyses. In this section some of the problems that one may encounter using the method of least squares fitting are briefly discussed.

One problem in model fitting is using the <u>wrong model</u>. For example, consider Fig. 2, visually the data appear to consist of two clusters of points. Within each of the clusters y decreases as x increases. If a straight line is fitted to such a combined data set, the line would go through the centers of the two groups but would predict y increasing with x. Checking the fit becomes more difficult, however, when using many independent variables to predict a dependent response variable due to human inability to visualize four (or more) dimensional space. In addition to various mathematical methods for checking the fit of a model, some of which are discussed later, a variety of two-dimensional plots can be used to insure that the fit is reasonable. A common practice in higher dimensions is to plot the residual value versus the predicted value. If the prediction is off by a larger amount for larger (or smaller) predicted values, such a plot would show the trend. Figure 3 gives another example of a wrong model; fitting curvilinear data with a straight line.

A second problem is created by what are called <u>outliers</u>. A data point is an outlier if in some sense it is separated by a considerable amount from the rest of the data. Figures 4 and 5 illustrate what can happen if a data set has an outlier. Figure 4 shows the body of data where the value of the dependent variable y tends to decrease with the value of the dependent variable x. When an outlier is added, a line of best fit has the form shown using the least squares method, rather than reflecting the trend of y decreasing with increasing x. Figure 5 shows a data point which is an outlier having an extreme value for the dependent variable y. It causes a sizeable shift in the curve for best fit. Other variations occur when using higher order curves for a model. Occasionally outlying values result from poor experimental technique or values

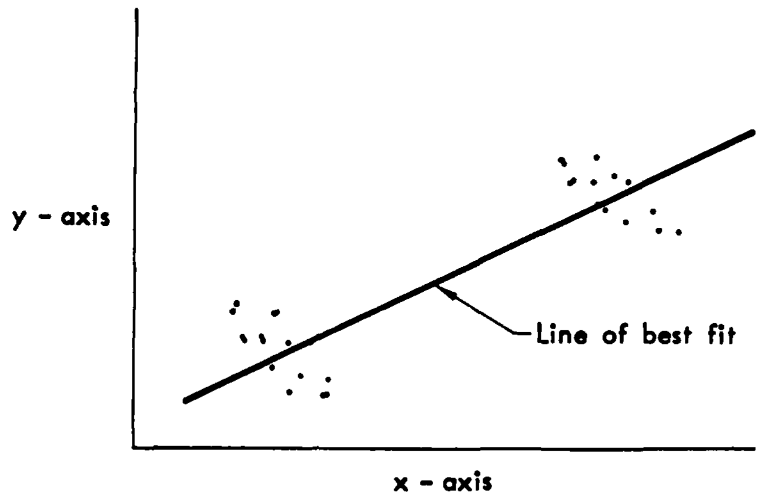


Fig. 2 Wrong model

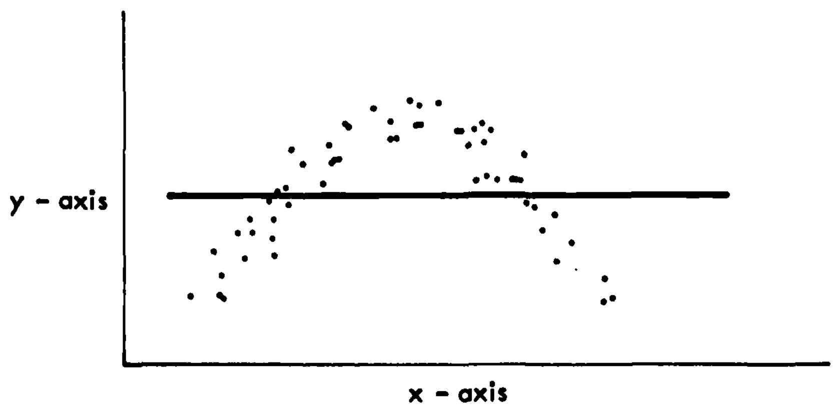


Fig. 3 Curvilinear data with straight line fit

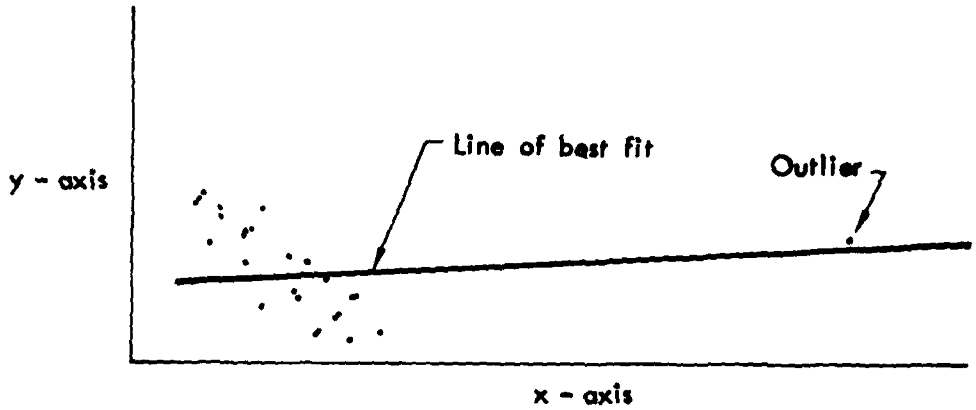


Fig. 4 One outlier

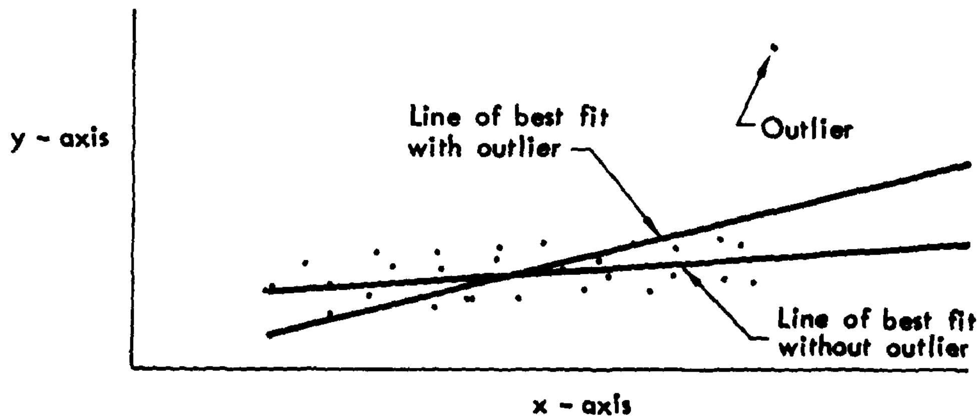


Fig. 5 Data with outlier

inappropriately measured. In other circumstances outliers may reflect some sort of transitional or phase effect. In model fitting one should consider the possibility of outliers and examine the affect that these data have on the model fit.

2-5 GOODNESS OF FIT

It is useful to have a measure of how well the fitted model explains the data. Suppose there are n data points and for each there is a dependent variable y, whose variability is to be explained; and a vector of independent variables x_i, which will be used to explain the y variability. After performing a least-squares fit to the data, how well has the explanation worked? Suppose for the moment that the independent variable x is not used to predict y and only a constant is to be used to predict the value for y. Equation 2 below shows how to pick the constant using the principle of least squares.

$$\text{minimum} \sum_{i=1}^{n} (y_i - C)^2 \tag{2}$$

That is, C is chosen to minimize the sum of squares of the deviations of y about the fixed constant value C. It can be shown that the value of C that minimizes the sum of squares is the mean of the sample data, or $\bar{y}$.

$$\bar{y} = \frac{Y_1 + \ldots + Y_n}{n} = \sum_{i=1}^{n} y_i/n \tag{3}$$

Now, if a dependence on the variable x is used, there will be some residual variability left even after fitting the model. The residual variability as defined above is given by eq. 4.

$$\sum_{i=1}^{n} (y_i - \hat{y}_i)^2 \tag{4}$$

The residual variability is the sum of squares of the deviations of residuals, the observed value y_i minus its predicted value $\hat{y}_i$. Starting with the

variability of eq. 2 where C is equal to $\bar{y}$ the variability left is given by eq. 4. Thus, the fraction of the variability not explained is given by eq. 5.

$$\sum_{i=1}^{n} (y_i - \hat{y}_i)^2 / \sum_{i=1}^{n} (y_i - \bar{y})^2 \qquad (5)$$

Hence, the fraction of the variability explained, called the multiple correlation coefficient is one minus this quantity. It is usually denoted by R^2 and can be written

$$R^2 = 1 - \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2} \qquad (6)$$

If the y_i values are not all the same so that the denominator is nonzero and if the family of curves being fitted contains the constant function, the value of R^2, lies between zero and one. This value will be equal to one if and only if the curve has fit the data exactly; that is, every single datapoint lies on the fitted model. This is almost never the case. The R^2 value will be zero only if the fitted model is a constant. Once again, this is usually not the case. Even if a "true" R^2 value is zero, (i.e. the family of predicted equations has no more predictive ability than a constant), experimental variability will usually allow some fitting of the data so that the observed R^2 will be nonzero. The multiple correlation coefficient R^2 is a measure of the success of the least squares fit -- it is the fraction of the variability in the dependent variable explained by the fit of the model.

2-6 NONLINEAR STATISTICAL MODELS

Some differences occur in the least-squares method when the unknown parameters are used linearly in the prediction of y and when they are not used linearly. A model is called linear if the "true" model is such that the

expected value of y is equal to a linear sum. Each term of the sum may involve an unknown coefficient times a know function of the independent variable x. Examples of this are given in eqs. 7 below where the a_i are unknown

$$y = a_0 + a_1 x \tag{7a}$$

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \tag{7b}$$

$$y = a_0 + a_1 e^x \tag{7c}$$

$$y = a_0 \sin x + a_1 \cos x + a_2 \sin 2x + a_3 \cos 2x \tag{7d}$$

$$y = a_0 e^x + a_1 xe^x + a_2 x + a_3 e^{-x} + a_4 xe^{-x} \tag{7e}$$

$$y = a_0 e^x + a_1 e^{x^2} + a_2 x + a_3 e^{-x} + a_4 e^{-x^2} \tag{7f}$$

In contrast when y is not a linear function of the a_i, the models are called nonlinear. Some examples are given in eqs. 8 below.

$$y = \exp (a_0 + a_1 x + a_2 x^2) + a_3 x \tag{8a}$$

$$y = a_0 + \sin (a_1 x) + \cos (a_2 x) \tag{8b}$$

$$y = a_0 + a_1 x^{a_2} + a_3 \exp(-a_4 x^2) \tag{8c}$$

Note that the mathematical model is linear when it is linear in the unknown coefficients but not necessarily a linear function of the independent variables x_i, which may be treated as a generalized vector.

There are some important differences between linear and nonlinear models. The least squares solution for linear models can be written explicitly. Except possibly for the numerical analysis problems of inverting a near-singular matrix, the least squares solution of a linear model may be found easily,

efficiently, and accurately. On the other hand, the nonlinear model is such that it is more difficult to find a solution. Except for a few very special cases, the variability is minimized by selectively varying the values of the parameters to reduce the residual sum of squares. The result leads to a local minimum; that is, if you move slightly in any direction by changing the parameter values, the value of the sum of squares becomes larger. This, however, does not assure that a global minimum over all possible values for the parameter was determined.

An illustration of a sum of squares for a nonlinear model with one parameter is shown in Fig. 6. A local minimum at θ_1 is not the overall minimum which is at θ_2. If one tries to find a nonlinear least squares solution by starting at θ_3, the answer would probably turn out to be θ_1; whereas if one starts at θ_4, the answer would turn out to be θ_2. In general, there is no known way to assure finding a true global minimum in nonlinear minimization. Having some idea of the correct value, however, will usually lead to the global minimum. In general, nonlinear optimization takes considerably more computational time and has more numerical analysis problems than do linear least square solutions.

2-7 THE DIFFERENCE BETWEEN PREDICTIONS USING MODEL FITTING AND THE DEVELOPMENT OF "TRUE" MODELS FOR A PHYSICAL SITUATION

To fit a mathematical model for purposes of prediction, it is not assumed that the model fitted is a "true" model with physically significant parameters. Rather, it is a model descriptive of the overall pattern of the data. Usually to develop a "true" model, there needs to be some physical understanding of the problem which gives the appropriate form of the model to be fitted. However for the purposes of prediction, a good fit to the data is usually satisfactory.

2-8 GUIDELINES FOR LEAST-SQUARES MODEL FITTING

(1) It is dangerous to extrapolate very far beyond the range of observed data for models fit for prediction purposes. This is not as true of models developed where it is known that the model is correct and has physical

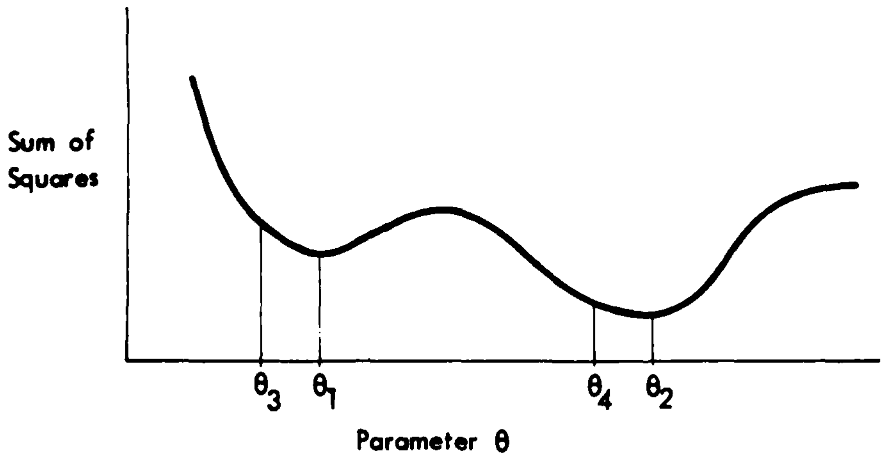


Fig. 6 Local minima at θ_1

significance. This is one of the advantages of the dimensional analysis approach. Since the dimensional analysis is developed by modeling the physics involved, by using similar experiments there is scientific reason to believe the prediction will hold up better than a purely empirical fit.

(2) The more complex the model, the more precarious is the extrapolation of predictions to other situations.

(3) Increasing the numbers of parameters is increasingly precarious using nonlinear-least-square models and can result in great numerical difficulties in finding the appropriate fit.

(4) Although theoretically one more observation than the number of parameters is needed for the least-square fit, in practice the number of observations for estimating the model should always be at least five (preferably a minimum of ten) per parameter used.

SECTION 3
DIMENSIONAL ANALYSIS FOR EXPLOSIVE CRATERING

An important aspect of cratering phenomena is the prediction of the crater resulting from a given explosive in a given soil or rock medium. Often direct experimentation is not possible. Alternatives include numerical simulation (e.g., Knowles and Brode, 1977; Cooper, 1977; Maxwell et al. 1973; Orphal, 1977; Swift, 1977 and others) or scaling the results of other experiments or tests. Frequently, results of small laboratory or field tests are scaled in some manner to predict the results of events many times larger in magnitude. It is this scaling application that is addressed in this section.

The objective in this section is to examine the exact conditions under which special scaling rules will hold and how the results depend on both the number and the choice of independent variables. The difference between using either the mass or the energy of the explosive is examined. It is shown that the choice is immaterial, except in those cases when other variables are assumed to be held constant. Under these conditions it is shown that varying the energy as opposed to varying the mass are two very different hypotheses, leading to different results. In the literature, differences in results, interpretations and uncertainties over the applicability of various scaling rules are many times due to a failure to distinguish between similar and nonsimilar experiments. This distinction is discussed and scaling of both types is considered.

3-1 BACKGROUND

Two scaling rules are well known. In the traditional "cube-root" scaling, all crater linear dimensions are assumed to vary with the cube-root of either the energy or the mass of the explosive. Consequently, the volume of the crater is proportional to the total energy or the mass of the explosive. Likewise, for "quarter-root" scaling, all crater linear dimensions are assumed to vary with the one-fourth root of the explosive energy or mass. This form is sometimes referred to as gravity-scaling (e.g., Gault and Wedekind, 1977).

Which, if either, of these two scaling rules is appropriate has been the subject of numerous papers dealing with cratering (Chabai, 1965, 1967, 1977; Crowley, 1970; Gault and Wedekind, 1977; Killian and Germain 1977; Vortman, 1968, 1977; and others). It is commonly thought that cube-root scaling should hold in a regime where soil material strength dominates the cratering mechanisms. Conversely, when the stresses in the cratering process are much larger than material strengths, quarter-root scaling is thought to hold. Chabai's work (1965) implies that whether the energy or the mass of an explosive is chosen as the size variable will determine which scaling rule will hold. Vortman (1968, 1977) does not distinguish between similar and nonsimilar experiments and consequently faults dimensional-analysis techniques as being unable to predict the different observed dependences of radius and depth upon yield for field-test results. White (1971) identified some of the shortcomings of various approaches to scaling in a summary paper comparing the methods up to that time.

3-2 FORMULATION

Consider the apparent crater volume V produced by a half-buried (zero depth of burst) spherical explosive in a geological medium. Non-zero depth of burst is considered later. The value of V is assumed to depend on the total explosive energy E, the explosive specific energy Q_e, the mass density of the explosive δ, the mass density of the soil ρ, the "strength" of the soil Y, and gravity g. For present purposes, the "strength" Y can be any material property with the units of pressure. While other variables can be identified which may influence V, their inclusion is not necessary for the arguments which follow in this section, as will be discussed later.

Other choices of variables to characterize the explosive can be interchanged with those in this list. For example, the mass W of the explosive is related to the total energy E by the specific energy per unit mass of explosive Q_e as follows:

$$W = E/Q_e. \tag{9}$$

The energy density per unit volume Q_v is given by

$$Q_v = \delta Q_e, \tag{10}$$

and the radius, r, of the explosive is determined by

$$(4\pi/3)\ r^3\ \delta = W. \tag{11}$$

If the stress quantity, Y is taken to be the bulk modulus K, the sonic velocity is given by $\sqrt{K/\rho}$.

In all cases, three independent variables are used to characterize the explosive. These include a measure of size, which can be one of W, E, or r, a measure of specific energy Q_e or energy density Q_v, and the mass density δ. (Nondimensional constants, such as the perfect gas constant of the explosive products, occur in common theories for detonating high explosives. However, the inclusion or omission of dimensionless constants will not affect any of the following results. Additional dimensional constants, which may be required for non-ideal explosives, can change the results as discussed in Section 3-5). Thus together with the soil properties of strength Y and density ρ and with gravity g, any dependent variable, such as volume V, is then a function of six independent variables. These variables involve independent units of mass, length, and time; hence, following standard methods of dimensional analysis, they can be reduced to a set of four dimensionless π-groups in a variety of ways; see, for example, Buckingham (1914), Baker et al. (1973) or Chabai (1977).

One possible combination, based upon using W, Q_e and δ to characterize the explosive, leads to a so-called <u>mass set:</u>

$$\pi_1 = \frac{V\rho}{W}, \tag{12a}$$

$$\pi_2 = \frac{g}{Q_e} \left(\frac{W}{\delta}\right)^{1/3}, \tag{12b}$$

$$\pi_3 = \frac{Y}{\delta Q_e}, \tag{12c}$$

$$\pi_4 = \frac{\rho}{\delta}. \tag{12d}$$

Another set based upon using E, Q_v and δ for the independent explosive properties will be called the <u>energy set</u>:

$$\bar{\pi}_1 = \frac{VQ_v}{E} , \tag{13a}$$

$$\bar{\pi}_2 = \frac{\rho g E^{1/3}}{Q_v^{4/3}} , \tag{13b}$$

$$\bar{\pi}_3 = \frac{Y}{Q_v} , \tag{13c}$$

$$\bar{\pi}_4 = \frac{\rho}{\delta} . \tag{13d}$$

Alternatively using E, Q_e and δ for the explosive gives a third set which can be referred to as the gravity set:

$$\bar{\bar{\pi}}_1 = V \left(\frac{\rho g}{E}\right)^{3/4} , \tag{14a}$$

$$\bar{\bar{\pi}}_2 = \frac{1}{Q_e} \left(\frac{g^3 E}{\delta}\right)^{1/4} , \tag{14b}$$

$$\bar{\bar{\pi}}_3 = \frac{Y}{\delta Q_e} , \tag{14c}$$

$$\bar{\bar{\pi}}_4 = \frac{\rho}{\delta} . \tag{14d}$$

Any one of these sets can be converted into either of the other two by simply multiplying products of powers of that set and using eqs. 9 through 11. Other combinations can also be developed.

For convenience of the reader, all the symbols and definitions are summarized in Table 1. Also included is notation which will be used in the analysis to follow.

3-3 SIMILARITY REQUIREMENTS

A given series of experiments are similar in the current context if they all have the same numerical value for each of π_1, π_2, π_3 and π_4, which

Table 1. Symbols and definitions used in the analysis.

V	apparent crater volume
W	mass of explosive charge
E	energy of explosive charge
r	equivalent radius of explosive charge
δ	mass density of explosive charge
Q_e	specific energy of explosive charge
Q_v	energy density of explosive charge
Y	"strength" of cratering medium, any material property with dimensions of pressure
ρ	density of cratering medium
g	gravity field strength
h	height of burst
α, β, γ	exponents introduced in eq. 27
$a = 1 - \alpha/3$, $b = \beta$	exponents defined in eq. 30
c, d	exponents introduced in eq. 39
π_i	"mass set"--dimensionless parameters defined in eqs. 12
$\bar{\pi}_i$	"energy set"--dimensionless parameters defined in eqs. 13
$\bar{\bar{\pi}}_i$	"gravity set"--dimensionless parameters defined in eqs. 14

implies that any other set of π-groups will also be constant for the series. (Note, a more general definition of similarity for cratering experiments given by Schmidt and Holsapple (1978a, 1980a) does not rest upon any assumptions about the list of independent variables.) Assuming that V is determined by the six independent variables given above, then π_1 must be determined solely by the remaining π_2 through π_4. Thus, if π_2 through π_4 each have the same value for different experiments, π_1 will also. From these sets of π-groups, certain scaling rules for similar experiments are commonly deduced by the following arguments.

From the mass set, since π_1 has the same value for such similar experiments,

$$V \propto W. \tag{15a}$$

Likewise, from the energy set, since $\tilde{\pi}_1$ is constant,

$$V \propto E. \tag{15b}$$

And from the gravity set, since $\bar{\bar{\pi}}_1$ is constant,

$$V \propto E^{3/4} . \tag{15c}$$

Equations 15a and 15b are called cube-root scaling, while eq. 15c is called quarter-root scaling because linear crater dimensions will then vary with W or E to those powers. Thus, it can be said that the first two sets of π-groups "imply" cube-root scaling, whereas the latter set "implies" quarter-root. From this point of view it appears that different scaling rules are obtained from different choices of dimensionless groups. This apparent paradox is, of course, specious and can be resolved by examining the exact conditions

under which each scaling rule applies. In each case, the scaling rule holds only for variations of the variables which preserve similarity. This means that each π-group must be held constant for the experiments under consideration. This is achieved in a different way for each of the above cases as is now shown.

For the mass set, V is proportional to W only if ρ is constant. In addition, π_2, π_3 and π_4 must be constant. This gives

$$\rho = \text{constant}, \tag{16a}$$

$$\delta = \text{constant}, \tag{16b}$$

$$\frac{g\ W^{1/3}}{Q_e} = \text{constant}, \tag{16c}$$

$$\frac{Y}{Q_e} = \text{constant}. \tag{16d}$$

For the energy set, V is proportional to E only if Q_v is constant. Also, $\bar{\pi}_2$, $\bar{\pi}_3$ and $\bar{\pi}_4$ are constant, requiring

$$Q_v = \text{constant}, \tag{17a}$$

$$\rho g\ E^{1/3} = \text{constant}, \tag{17b}$$

$$Y = \text{constant}, \tag{17c}$$

$$\frac{\rho}{\delta} = \text{constant}. \tag{17d}$$

For the gravity set, V is proportional to the three-fourths power of E only if ρg is constant. This, in addition to constant values of $\bar{\bar{\pi}}_2$, $\bar{\bar{\pi}}_3$ and $\bar{\bar{\pi}}_4$, gives

$$\rho g = \text{constant}, \tag{18a}$$

$$\frac{\rho}{\delta} = \text{constant}, \tag{18b}$$

$$g^{3/4}\ E^{1/4} \propto Q_e\ \delta^{1/4}, \tag{18c}$$

$$\frac{Y}{\delta Q_e} = \text{constant}. \tag{18d}$$

These three sets of conditions are different and prescribe different ways to preserve similarity involving changes in gravity, charge type and size, and medium properties. Since they relate to different ways to preserve similarity they each give a different dependence of volume on energy or mass.

Suppose similarity between experiments conducted in a fixed soil type is desired. In this special case, both ρ and Y are constant for those experiments, and it is assumed that Y is nonzero. Further restrictions on experimental conditions can now be written. For the mass set, the similarity conditions that would produce cube-root scaling are (from eqs. 16a through 16d)

$$Q_e = \text{constant}, \tag{19a}$$

$$\delta = \text{constant}, \tag{19b}$$

$$gW^{1/3} = \text{constant}. \tag{19c}$$

Likewise for the energy set, the similarity conditions that would produce cube-root scaling are (from eqs. 17a through 17d)

$$Q_v = \text{constant}, \tag{20a}$$

$$\delta = \text{constant}, \tag{20b}$$

$$gE^{1/3} = \text{constant}. \tag{20c}$$

In contrast, for the gravity set, eqs. 18a through 18d reduce to the following similarity conditions that would give quarter-root scaling

$$Q_e = \text{constant}, \tag{21a}$$

$$E = \text{constant}, \tag{21b}$$

$$\delta = \text{constant}, \tag{21c}$$

$$g = \text{constant}. \quad (21d)$$

Thus, similarity conditions leading to cube-root scaling from either the mass set or the energy set in a given material are possible and require using the same explosive, but gravity must be varied. At constant gravity, similarity conditions leading to cube-root scaling cannot be satisfied by varying size. From the gravity set, it can be concluded that there are no two different experiments satisfying the similarity requirements which lead to quarter-root scaling for experiments using a common explosive in a given material with nonzero strength (note in particular equation 21b). Hence, quarter-root scaling among similar experiments could only be realized by varying both the soil properties and the explosive properties. It should be noted that these statements do not imply that a choice of the gravity set rules out similar experiments in their entirety in a fixed material and with a fixed explosive. Only experiments that must give quarter-root scaling are eliminated and are impossible (i.e., eq.14a, $\bar{\bar{\pi}}_1$ = constant). Regardless of the choice of π-groups, similar experiments in a fixed material with a fixed explosive are only possible if gravity is varied as charge size is varied so that $gE^{1/3}$ is constant, and the volume must vary as the cube-root of energy or mass.

In the event that the soil has zero strength Y, other possibilities for similarity between experiments exist. In this case, restrictions based upon the material-strength π-group are not present. Hence, the mass set allows similar experiments with cube-root scaling of size in a given strengthless material whenever

$$\delta = \text{constant}, \quad (22a)$$

$$gW^{1/3}\, Q_e^{-1} = \text{constant}, \quad (22b)$$

whereby a fixed explosive is not required. However, for a fixed explosive, g must be varied.

For the energy set, with a strengthless medium, the results are the same as for a medium with strength, given above by eqs. 20a-20c.

For the gravity set, eliminating the strength term gives, for a fixed medium,

$$g = \text{constant}, \tag{23a}$$

$$E^{1/4}\, Q_e^{-1} = \text{constant}, \tag{23b}$$

$$\delta = \text{constant}. \tag{23c}$$

Quarter-root scaling is possible in fixed soil having zero strength, but only with different explosives.

All of these cases are summarized in Table 2. It is seen that the reason that different scaling rules are generated from different choices of π-groups is because they require different constraints to achieve similarity. Quarter-root scaling is only allowed among experiments with explosives having different specific energy, as specified by eq. 23b. For a series of experiments with both medium and explosive type fixed, similarity requires variations in gravity; and cube-root scaling results. This can be proved from any choice of dimensionless parameters.

3-4 SCALING NONSIMILAR EXPERIMENTS

Scaling based upon experiments that are <u>nonsimilar</u> is determined by the functional relationship among π-groups given by

$$\pi_1 = F\,(\pi_2, \pi_3, \pi_4), \tag{24a}$$

$$\bar{\pi}_1 = \bar{F}\,(\bar{\pi}_2, \bar{\pi}_3, \bar{\pi}_4) \tag{24b}$$

$$\bar{\bar{\pi}}_1 = \bar{\bar{F}}\,(\bar{\bar{\pi}}_2, \bar{\bar{\pi}}_3, \bar{\bar{\pi}}_4). \tag{24c}$$

For a given medium and explosive, the last two π-groups are constant in each set, and the first π-group reduces to a function of only the second:

$$\pi_1 = G\,(\pi_2), \tag{25a}$$

Table 2. Similarity requirements for cratering experiments.

Case Number	Assume that volume V depends upon	The choice of the dimensionless groups	shows that V is proportional to	under conditions of constancy of each of	which, for a given medium with strength, requires constancy of each of	or for a given strengthless medium, requires constancy of each of
1.0	$W, Q_e, \delta, \rho, Y, g$	$\pi_1 = \frac{V\rho}{W}$		ρ	-	-
	(Referred to as the mass set where $E = WQ_e$)	$\pi_2 = \frac{g}{Q_e}\left(\frac{W}{\delta}\right)^{1/3}$	W (cube-root scaling)	$\frac{gW^{1/3}}{Q_e}$	$gW^{1/3}$	$gW^{1/3}\ Q_e^{-1}$
		$\pi_3 = \frac{Y}{\delta Q_e}$		$\frac{Y}{Q_e}$	Q_e	-
		$\pi_4 = \frac{\rho}{\delta}$		δ	δ	δ
2.0	$E, Q_v, \delta, \rho, Y, g$	$\bar{\pi}_1 = \frac{VQ_v}{E}$		Q_v	Q_v	Q_v
	(Referred to as the energy set)	$\bar{\pi}_2 = \frac{\rho g E^{1/3}}{Q_v^{4/3}}$	E (cube-root scaling)	$\rho g E^{1/3}$	$gE^{1/3}$	$gE^{1/3}$
		$\bar{\pi}_3 = \frac{Y}{Q_v}$		Y	-	-
		$\bar{\pi}_4 = \frac{\rho}{\delta}$		$\frac{\rho}{\delta}$	δ	δ
3.0	$E, Q_e, \delta, \rho, Y, g$	$\bar{\bar{\pi}}_1 = V\left(\frac{\rho g}{E}\right)^{3/4}$		ρg	g	g
	(Referred to as the gravity set).	$\bar{\bar{\pi}}_2 = \frac{1}{Q_e}\left(\frac{g^3 E}{\delta}\right)^{1/4}$	$E^{3/4}$ (quarter-root scaling)	$\left(\frac{g^3 E}{\delta}\right)^{1/4} \frac{1}{Q_e}$	E (which rules out similar experiments)	$E^{1/4}\ Q_e^{-1}$
		$\bar{\bar{\pi}}_3 = \frac{Y}{\delta Q_e}$		$\frac{Y}{\delta Q_e}$	Q_e	-
		$\bar{\bar{\pi}}_4 = \frac{\rho}{\delta}$		$\frac{\rho}{\delta}$	δ	δ.

$$\bar{\pi}_1 = \bar{G}\,(\bar{\pi}_2), \tag{25b}$$

$$\bar{\bar{\pi}}_2 = \bar{\bar{G}}\,(\bar{\bar{\pi}}_2). \tag{25c}$$

These functions cannot be deduced from dimensional analysis arguments, but must be determined by experiments or other means; determination of any one function will determine the others. Additional assumptions will now be introduced which lead to scaling rules relating nonsimilar experiments. These assumptions are that certain variables do not influence the volume V (at least over some given range).

Consider the above energy set for the general case given by eq. 24b:

$$\bar{\pi}_1 = \frac{VQ_v}{E} = \bar{F}\left[\frac{\rho g\, E^{1/3}}{Q_v^{\;4/3}}, \frac{Y}{Q_v}, \frac{\rho}{\delta}\right]. \tag{26}$$

For some range of interest of the independent variables, assume that the function $\bar{F}$ in eq. 26 can be represented by a product of powers having the form

$$\bar{\pi}_1 = k\, \bar{\pi}_2^{\,-\alpha}\, \bar{\pi}_3^{\,-\beta}\, \bar{\pi}_4^{\,-\gamma}, \tag{27}$$

where k is a constant of proportionality. (Whenever the variables are bounded away from zero, a linear Taylor series approximation of log π_1 as a function of log π_2, log π_3 and log π_4 will give this form. Thus this form can always be assumed locally, except in the case of zero strength, which must be treated separately. Whether or not such an approximation holds over any large range of interest is actually immaterial; the restrictions on the exponents to be derived below are bounds on the slopes of the function on a log-log plot. The motivation for this power-law form is its common usage for crater scaling rules.)

Using the definitions of the π-groups gives

$$\frac{VQ_v}{E} = k \left[\frac{\rho g E^{1/3}}{Q_v^{4/3}}\right]^{-\alpha} \left[\frac{Y}{Q_v}\right]^{-\beta} \left[\frac{\rho}{\delta}\right]^{-\gamma} \tag{28}$$

which can be rearranged giving

$$V \propto E^{a} g^{3(a-1)} Q_v^{3-4a+b} Y^{-b} \tag{29}$$

with

$$a = 1 - \alpha/3, \tag{30a}$$

$$b = \beta, \tag{30b}$$

and where the density terms have been omitted, since they do not contribute to the analysis to follow.

Fundamental constraints on scaling brought about by restrictions on the exponents follow from assumptions on the sign of the volume variation due to changes in each independent variable. In particular, it is now assumed that, in eq. 29, for all other variables held fixed, the volume will not decrease (i.e., it will increase or remain constant) as

- the total energy increases,
- gravity decreases,
- the energy density Q_v decreases, or
- the strength Y decreases.

These assumptions are supported by experimental evidence and energy balance arguments. As the energy increases, more energy is available to do work. Both the gravity and material strength require work to be done during the excavation. As the energy density decreases, the transmitted stresses are lower, less energy is lost to internal energy in the cratered medium, and the overall process efficiency increases. This point has been made by Burton, et al. (1974). Note that all other variables in eq. 29 are assumed fixed in these assumptions so that, for example, the assumption that V increases as Q_v decreases is for fixed energy, not fixed mass.

The four assumptions listed above imply that

$$a \geq 0 \tag{31a}$$

$$b \geq 0 \tag{31b}$$

$$3(a - 1) \leq 0 \tag{31c}$$

$$3 - 4a + b \leq 0. \tag{31d}$$

Equations 31b and 31d can be combined as

$$0 \leq b \leq 4a - 3 \tag{32a}$$

from which

$$a \geq 3/4 \tag{32b}$$

which, used with eq. 31c, gives

$$3/4 \leq a \leq 1. \tag{33}$$

Using eq. 9 and 10 in eq. 29, gives

$$V \propto W^a \, Y^{-b} \, g^{3(a-1)} \, Q_e^{3(1-a)+b} \tag{34}$$

Thus the above bounds provide limiting dependences upon both charge mass and on energy. The case of a = 1 is cube-root scaling, while a = 3/4 gives quarter-root scaling. Therefore, the above assumptions require that all scaling be bounded by these two extremes. The exponent b is restricted by eq. 32a and depends upon the value of a. For the limiting case of cube-root scaling, when a = 1,

$$0 \leq b \leq 1. \tag{35}$$

This permissible range on b is reduced as the value of a decreases; and for quarter-root scaling, a = 3/4, b must be zero and there can be no strength dependence.

Other special cases occur when the exponents take on particular values. Note that certain combinations of the independent variables can be identified as stress quantities. For example, the energy density $Q_v = \delta Q_e$ determines the Chapman-Jouguet (C-J) pressure of the explosive. Thus, $\bar{\pi}_3$ is proportional to the ratio of the medium strength to the C-J pressure. Likewise $\bar{\pi}_2$ (using eqs. 9 and 10) is

$$\bar{\pi}_2 = \frac{\rho g\, E^{1/3}}{Q_v^{\,4/3}} = \left(\frac{4\pi}{3}\right)^{1/3} \frac{\rho g r}{Q_v} \tag{36}$$

which is proportional to the ratio of the lithostatic pressure at the charge base to the explosive C-J pressure.

If the lithostatic pressure is very much smaller than either the soil strength Y or the C-J pressure, perhaps $\bar{\pi}_1$ is independent of $\bar{\pi}_2$ and the soil strength dominates crater formation mechanics. (Holsapple and Schmidt (1979) and Holsapple (1979a) have observed such a regime for cohesive materials. The material strength Y used here can be taken to be the cohesion c in those papers.) If $\bar{\pi}_1$ is assumed to be independent of $\bar{\pi}_2$ and hence independent of gravity, the exponent a must be equal to one ($\alpha = 0$); cube-root scaling will then hold, and the volume V is given by

$$V \propto E Y^{-b} Q^{b-1} \tag{37}$$

Assume alternatively that the soil strength Y can be neglected. Then $b = \beta = 0$ and the exponent a is only restricted by eq. 33, allowing any scaling between cube-root and quarter-root. If in addition, it is assumed that the volume is independent of the energy density Q_v, the exponent of Q_v in eq. 29 must be zero. This along with b = 0 gives a = 3/4, resulting in quarter-root scaling.

Other combinations of dependence or independence of the variables under consideration exist and lead to different results. A summary of these results is given in Table 3 for each of the three choices of π-group sets given above. These were derived under the assumption that the functional dependences indicated in eqs. 24a through 24c could be represented locally by a product of powers of the pertinent π-groups, as was done for the energy set in eq. 27. (It might be noted that for dry sand such a power law dependence of π_1 on the π_2 variable has been shown to be valid both for explosive and for impact cratering over a total range of eighteen orders of magnitude in scaled energy by Schmidt and Holsapple, (1978a, 1978b, 1980a), Schmidt et al. (1979), Schmidt (1980) and by Gault and Wedekind (1977). For other materials such a power law may hold only locally as shown by Holsapple (1979a) and Holsapple and Schmidt (1979).

The volume dependence (or independence) is then considered for g, Y and either Q_e or Q_v. Restrictions on the exponents given are based on the same assumptions given above following eq. 30. Namely, that for all other variables held fixed, the volume V must not decrease as g or Y decrease or as E or W increase. Also for fixed energy ($E = Q_e W$) and all other variables held fixed, the volume must not decrease as Q_e (or Q_v) decreases. In all these cases, five independent variables are held fixed, including ρ and δ.

The assumption that the volume is independent of Q_e at fixed W, included in cases 1.3, 1.4, and 1.5 (Table 3) leads to the further requirement that the volume is independent of the total charge energy at fixed charge mass. This is contrary to experimental evidence (Schmidt and Holsapple, 1978a, 1980a). Nevertheless, these cases are included, since they relate to conditions implicitly assumed by Chabai (1965) and others using mass scaling. If Q_e is omitted from the initial set of variables, a dependence on W alone is not a meaningful assumption, as discussed by Divoky (1966).

The results presented in Table 3 can be shown graphically as a region of permissible values for the various exponents governing the functional dependences. Consider case number 3.0 of Table 3, shown in Fig. 7 as the specific-energy exponent versus the energy exponent. All permissible pairs of these exponents are contained within the bounds shown. Figure 8 is a permutation of the same information, with the material-strength exponent shown

Table 3. Summary of scaling laws relating nonsimilar cratering experiments.

Case Number	Crater Volume V Depends on These Specified Variables	Crater Volume V Assumed Independent of	Resulting Scaling Rule for Crater Volume V, V ∝:	Restrictions on Exponents	Remarks Concerning Crater Volume Dependence Upon Charge Energy (or Mass)
1.0		None	$W^aY^{-b}g^{3(a-1)}Q_e^{3(1-a)+b}$	$3/4 \le a \le 1$, $0 \le b \le 4a-3$	bounded by quarter-root and cube-root (quarter-root implies b=0)
1.1	W, Q_e, δ, ρ, Y, g	Y	$W^ag^{3(a-1)}Q_e^{3(1-a)}$	$3/4 \le a \le 1$, $b = 0$	bounded by quarter-root and cube-root
1.2	(referred to as	g	$W^1Y^{-b}Q_e^b$	$a = 1$, $0 \le b \le 1$	results in cube-root scaling
1.3	the mass set	Q_e*	$W^1g^0Y^0$	a=1, b=0	results in cube-root scaling*
1.4	where E = WQ_e)	Y, Q_e*	W^1g^0	a=1, b=0	results in cube-root scaling*
1.5		g, Q_e*	W^1Y^0	a=1, b=0	results in cube-root scaling*
1.6		g, Y	$W^1Q_e^0$	a=1, b=0	results in cube-root scaling
2.0		None	$E^aY^{-b}g^{3(a-1)}Q_v^{3-4a+b}$	$3/4 \le a \le 1$, $0 \le b \le 4a-3$	bounded by quarter-root and cube-root (quarter-root implies b=0)
2.1	E, Q_v, δ, ρ, Y, g	Y	$E^ag^{3(a-1)}Q_v^{3-4a}$	$3/4 \le a \le 1$, $b = 0$	bounded by quarter-root and cube-root
2.2	(referred to as	g	$E^1Y^{-b}Q_v^{b-1}$	$a = 1$, $0 \le b \le 1$	results in cube-root scaling
2.3	the energy set)	Q_v	$E^ag^{3(a-1)}Y^{3-4a}$	$3/4 \le a \le 1$, $b=4a-3$	bounded by quarter-root and cube-root
2.4		Y, Q_v	$E^{3/4}g^{-3/4}$	a=3/4, b=0	results in quarter-root scaling
2.5		g, Q_v	E^1Y^{-1}	a=1, b=1	results in cube-root scaling
2.6		g, Y	$E^1Q_v^{-1}$	a=1, b=0	results in cube-root scaling
3.0		None	$E^aY^{-b}g^{3(a-1)}Q_e^{3-4a+b}$	$3/4 \le a \le 1$, $0 \le b \le 4a-3$	bounded by quarter-root and cube-root (quarter-root implies b=0)
3.1	E, Q_e, δ, ρ, Y, g	Y	$E^ag^{3(a-1)}Q_e^{3-4a}$	$3/4 \le a \le 1$, $b = 0$	bounded by quarter-root and cube-root
3.2	(referred to as	g	$E^1Y^{-b}Q_e^{b-1}$	$a = 1$, $0 \le b \le 1$	results in cube-root scaling
3.3	the gravity set)	Q_e	$E^ag^{3(a-1)}Y^{3-4a}$	$3/4 \le a \le 1$, $b=4a-3$	bounded by quarter-root and cube-root
3.4		Y, Q_e	$E^{3/4}g^{-3/4}$	a=3/4, b=0	results in quarter-root scaling
3.5		g, Q_e	E^1Y^{-1}	a=1, b=1	results in cube-root scaling
3.6		g, Y	$E^1Q_e^{-1}$	a=1, b=0	results in cube-root scaling

*Included for completeness only, as it is physically unreasonable and inconsistent with observations (see text and Figure 9).

$$V \propto E^{a} Y^{-b} g^{3(a-1)} Q_e^{3-4a+b}$$

(CASE 3.0)

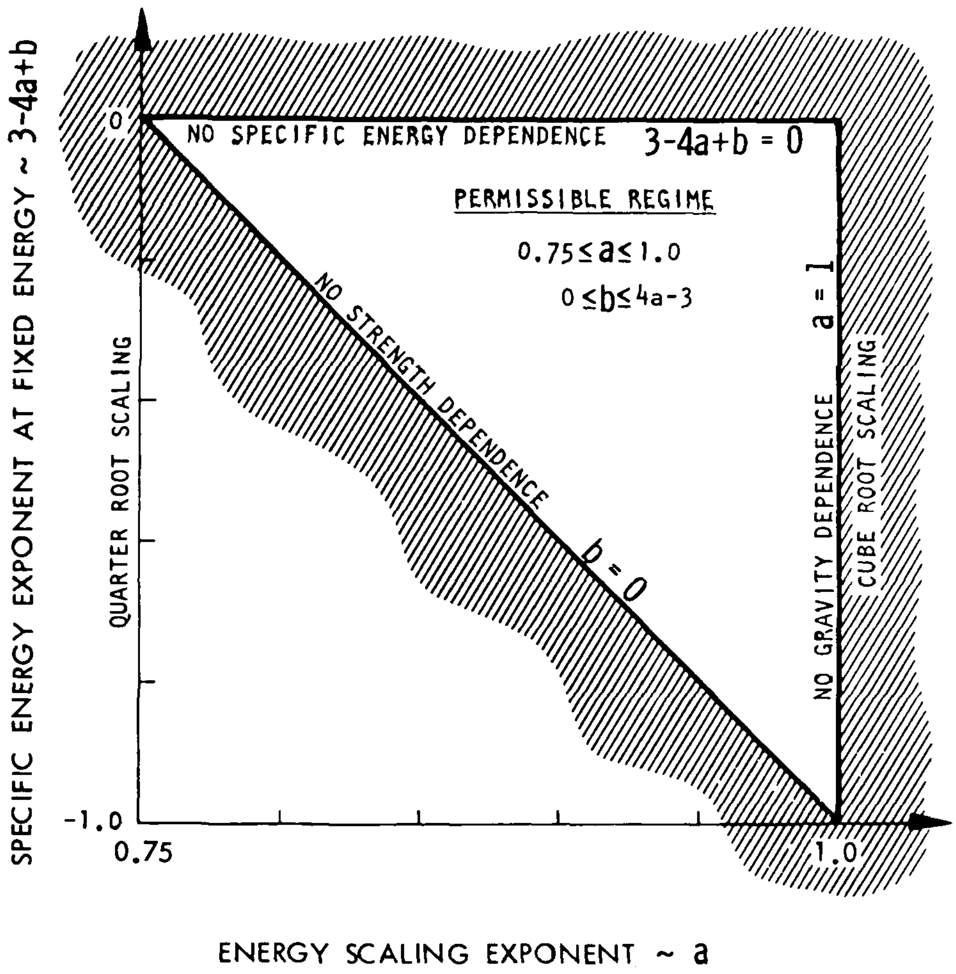


Fig. 7 Admissible bounds on scaling exponents shown on plot of the energy exponent a versus the specific energy exponent at fixed energy. As can be seen, quarter-root scaling, $a = 3/4$, is only permissible under the condition of no specific energy dependence, $3-4a+b = 0$. Note different horizontal and vertical scales.

$$V \propto E^a \, Y^{-b} \, g^{3(a-1)} \, Q_e^{\,3-4a+b}$$

(CASE 3.0 ALTERNATIVE PLOT)

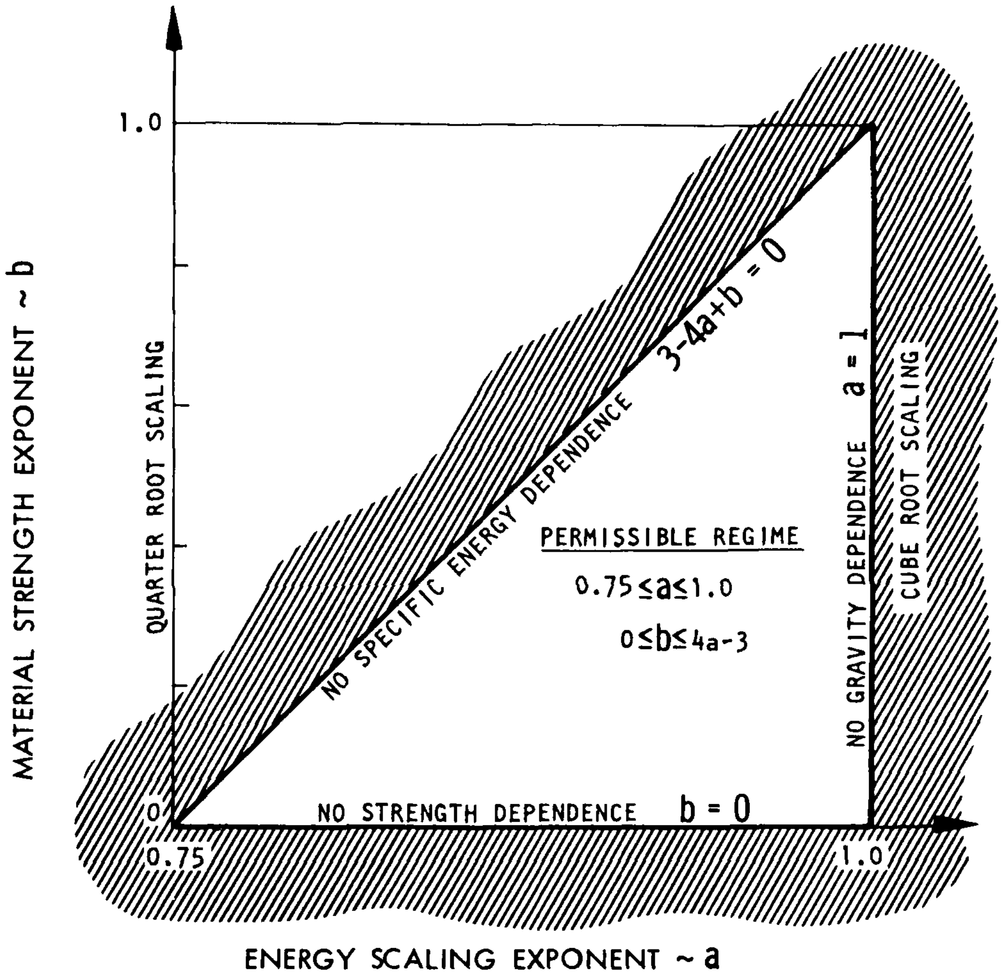


Fig. 8 Admissible bounds on scaling exponents shown on plot of the energy exponent a versus the material strength exponent b. Note different horizontal and vertical scales.

as the ordinate. These figures clearly show that quarter-root scaling requires independence from both strength and specific-energy effects.

Figure 9 shows another example corresponding to case number 1.0 of Table 3, where the mass of the explosive is the independent variable for charge size. This illustrates the impossibility of having no dependence upon specific energy for fixed explosive mass, unless there is also no gravity and no strength dependence. As mentioned above, it is important to recognize the distinction between this assumption and the assumption of no dependence on energy density for fixed explosive energy. These two conditions give two entirely distinct lines in Fig. 9.

The following statements are based on the results summarized in Table 3 and in Figs. 7 through 9. Although the apparent crater volume was used as a representative crater-dependent variable, these results also apply to crater radius, crater depth, true crater volume, and any other size variable of interest that is assumed to be restricted by the given assumptions following eq. 30 listed above. These conclusions apply to the scaling of all nonsimilar experiments.

1) Cratering dependence on energy is bounded by cube-root and quarter-root scaling rules for fixed explosive type in a given medium.
2) The additional assumption of no gravity dependence gives cube-root scaling. Subsequent dependences on Y and Q_e (or Q_v) are related as shown in cases 1.2, 2.2, and 3.2 of Table 3.
3) No dependence on strength Y does not imply quarter-root scaling. A further requirement of no dependence on Q_e or Q_v at fixed energy is necessary for quarter-root scaling to hold as shown by eq. 29.
4) The assumption of no dependence on Q_e, when using the mass W as a measure of explosive size, is entirely different from assuming no dependence of Q_e when using the energy E as the explosive-size measure. Furthermore, the former case is contrary to any known experimental observations.

$$V \propto W^a \, Y^{-b} \, g^{3(a-1)} \, Q_e^{3(1-a)+b}$$

(CASE 1.0)

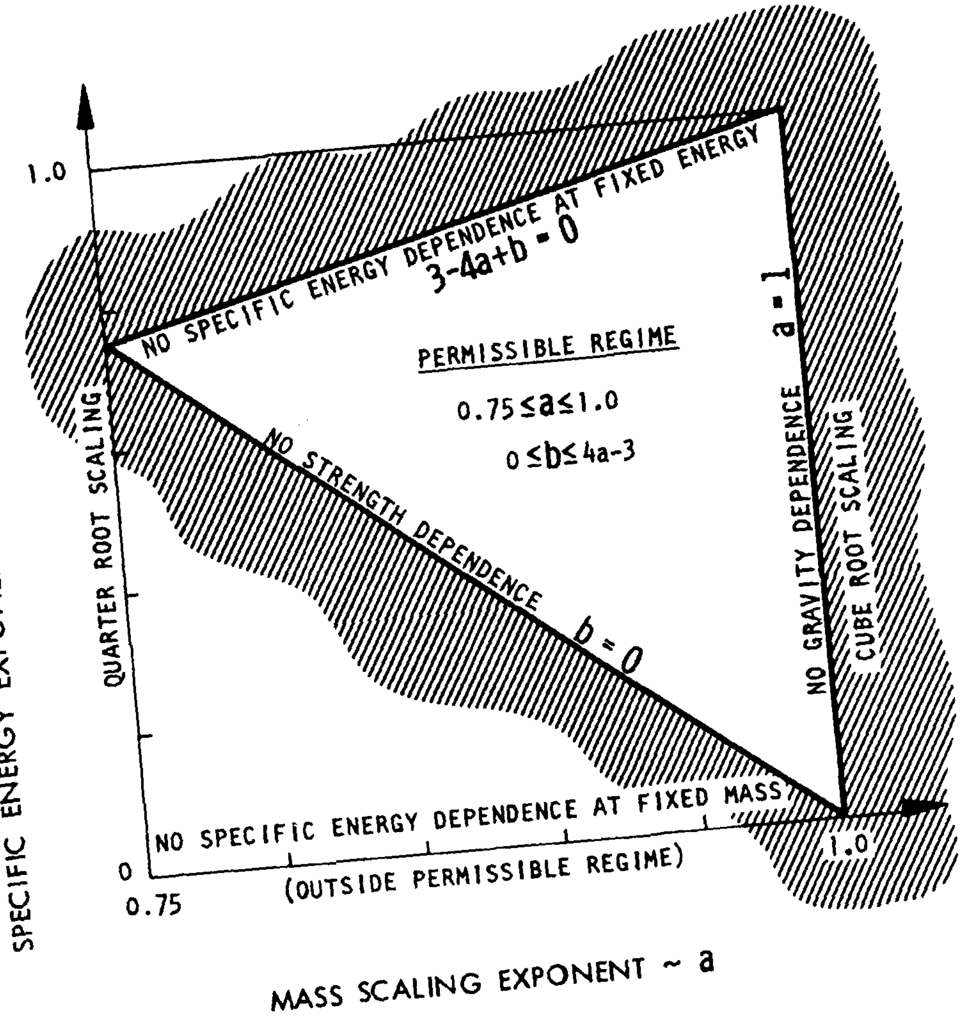


Fig. 9 Admissible bounds on scaling exponents shown on plot of the mass exponent a versus the specific energy exponent at fixed mass. Note different horizontal and vertical scales.

5) In every case, the assumption of no dependence on any two of the variables Y, g and either Q_e or Q_v produces a specific relationship between the energy dependence and the remaining variable.

Certain converse statements can also be made as follows:

6) If cube-root scaling holds, then the volume V is independent of gravity g.
7) If quarter-root scaling holds, then the volume is independent of not only the strength Y but also of the specific energy Q_e and of the energy density Q_v at fixed energy. No dependence on the strength is expected for certain size regimes; however, no dependence on the specific energy is less certain.

This last statement may explain why quarter-root scaling is not observed in practice. As identified in the foregoing discussion, three stress quantities are included in the variables: the explosive C-J pressure, the lithostatic pressure, and the strength of the medium. Even when the strength of the medium can be ignored, the ratio of the other two can affect the results and the scaling need not be quarter-root.

3-5 SCALING GENERALIZATIONS

Some of the differences between the results given here and those of other authors are due to the choice and the number of independent variables on which the crater characteristics are assumed to depend. For this reason, it is important to determine exactly how restrictive the choice is, and if the inclusion of additional variables would change the results.

The final crater formed by a given explosive is assumed to be determined by the geometry and the initial conditions of the problem and by the constitutive equations that describe the behavior of each of the three media: the explosive, the cratering medium, and the overlying air. For a half-buried spherical charge in an infinite half-space of soil, the only geometric parameter is the charge radius, which is implicitly included in the above list of

variables. For buried or suspended charges one must include the depth or height of burst dimension. The initial conditions include the initial densities and pressures in the various media. Therefore, the atmospheric pressure and the density of the air could be added to the list of independent variables. Remaining variables would be those related to constitutive properties of each of the three media. Many can be suggested, depending on the complexity of the equations that fully describe the actual media behavior.

As a first generalization then, a more general constitutive description of the cratering medium is considered. The crater excavation problem is basically mechanical in nature. A very general mechanical constitutive equation is one that relates the stress tensor at each point in the material, at a given time, to the entire history of the deformation in some neighborhood of that point. Within this context, the description of a material can depend on, at most, the three independent dimensions of stress, time, and length. A subclass of such materials called "simple materials" by Truesdell and Noll (1965), has the stress at a point determined by the entire past history of the strain at that point. These materials have no inherent length dimensions, but do have, at most, units of stress and time. Dimensions of time can be eliminated in the case of rate-independent behavior. This reduction still includes all types of nonlinear elastic, rate-independent plasticity, fracture, porosity, and many other diverse and complicated behaviors. For this group, all material constants will be either dimensionless or have units of stress. Other than the dimensionless π-groups given above, the only additional π-groups resulting from these constitutive equations would be ratios of each of those with stress units to the primary quantity Y and any dimensionless constants for the material as π-groups themselves.

For example, suppose a bulk modulus K of the medium was included. Then, in addition to the previous π-group, a new group

$$\pi_5 = \frac{K}{Y} \tag{38}$$

would occur. Additional material constants with dimensions of pressure, call them Y_1, Y_2,... would add additional such groups. Then eq. 28 would include powers of these π-groups, so that eq. (29) would become

$$V \propto E^a \, g^{3(a-1)} \, Q_v^{3-4a+b} \, Y^{-(b+c+d+\ldots)} \, K^c \, Y_1^d \qquad (39)$$

in terms of the original constants a and b and new powers c, d, etc.

Now, for a fixed medium, it is again plausible to assume that the volume will not decrease as

- E increases at fixed g and Q_v
- g decreases at fixed E and Q_v
- Q_v decreases at fixed E and g.

Therefore, from eq. 39

$$a \geq 0 \qquad (40a)$$

$$3(a-1) \leq 0 \qquad (40b)$$

$$3-4a+b \leq 0 \qquad (40c)$$

which gives

$$\frac{3}{4} + \frac{b}{4} \leq a \leq 1 \qquad (40d)$$

Hence, the scaling rule is bounded above by cube-root scaling, but cannot be proven to be bounded below by quarter-root scaling without further assumptions on the coefficient b.

Consider the special case with only the one additional variable, the bulk modulus K. It may be plausible to assume that, among various cratering materials, the volume will not decrease as

- The strength Y decreases at fixed K

or

- The bulk modulus K decreases at fixed Y.

The former assumption is identical to that made earlier. The second has been suggested by Chabai (1977). In this case,

$$b + c \geq 0 \tag{41a}$$

$$c \leq 0 \tag{41b}$$

so that

$$b \geq 0 \tag{41c}$$

and thus eq. 40d gives

$$\frac{3}{4} \leq a \leq 1, \tag{41d}$$

and quarter-root again becomes the lower bound. Conversely, it is true that there can be no dependence on either medium strength property Y or K, yet quarter-root scaling need not hold. If quarter-root scaling does hold, then eq. 40d and eq. 41c are compatible only if b = 0, eqs. 41a and 41b then give c = 0, and the volume must be independent of both material properties Y and K, as well as the explosive energy density Q_v.

If the additional parameters Y_1, Y_2, ... etc., are included, an assumption that the volume would not decrease as any one of them decreased, for all others fixed, would be sufficient to give the same result (41d). Whether this assumption was plausible would depend on the specific case.

All of the above results depend strongly on the assumption that the materials in question can be regarded as both rate- and scale-independent. In materials such as rock, there is evidence that the characteristic strength measures decrease as the size of the explosive increases and the resulting crater also increases. This is attributed to the increased influence of fault planes and crack nucleation sites as the size scale increases. As noted by Chabai (1977) both this size effect and also rate effects can give scaling that is in excess of cube-root (a > 1). This probably explains the results of Vortman (1977) where the exponent a is in excess of 1.0. However, it is not expected that this trend would continue to ever-increasing sizes; see Schmidt (1980).

Additional variables that define the initial state of the overlying air may include the atmospheric pressure P_0. Indeed, there is evidence that for buried bursts, increased P_0 gives decreased volume for all other variables held fixed, although this effect is not observed at zero depth of burst (Herr, 1970). Recent experiments on impact cratering show results analagous to those for buried explosives (Holsapple, 1979b). The atmospheric pressure has, of course, pressure dimensions and therefore would occur in the theory exactly as one of the material constants just considered for the soil. Furthermore, if the sign of the variation of the volume is assumed to be the same for pressure P_0 as that just considered for both Y and K of the soil, there is no need to repeat the analysis, eq. 41d again holds. (In contrast, for above ground bursts, recent data of the authors (Schmidt and Holsapple, 1980b) implies decreased volume for decreased pressure P_0. This is attributed to the decreased coupling of the energy to the ground in this case.)

The last important variable to be considered is the depth or height of the explosive for nonsurface-burst conditions. The additional π-group can be taken to be

$$\pi_6 = \frac{h}{r} \propto h \left(\frac{\delta}{W}\right)^{1/3} \tag{42}$$

where h is the distance from the ground surface to the charge center.

Inclusion of this variable will not affect any of the above arguments. It is only necessary that it be constant throughout. Consequently, all restrictions on scaling rules that have been derived at least insofar as the stated assumptions are valid, hold for any fixed value of h/r.

3-6 DISCUSSION OF SCALING RESULTS

A discussion of the relation of these results to those of some previous references is appropriate. In a paper dealing with scaling of crater dimensions, Chabai (1965) omits the specific energy Q_e from his list of independent variables. He then concludes: "When gravity field strength is considered significant and is included in the dimensional analysis, crater dimensions are scaled by cube-root rules with mass-gravity scaling and by

fourth-root rules with energy-gravity scaling." The implication that the choice of π-groups can lead to different resuts is a consequence of omitting the specific energy Q_e and by a failure to distinguish between the scaling of similar and nonsimilar experiments. Divoky (1966) correctly noted the effect of the omission of Q_e. Chabai (1965) also considered scaling for nonsimilar experiments, without explicitly noting that such an extension requires an assumption of independence of certain π-groups.

In a later paper, Chabai (1977) includes the specific energy Q_e in an ancillary π-group. He recognized the requirement that specific energy must be varied to preserve similarity; however, he states: "From dimensional analysis we obtain cube-root scaling rules or quarter-root scaling rules depending on whether or not gravity is considered unimportant or important in cratering." The present analysis gives results that agree with only the first part of this statement, regarding cube-root scaling. It is not true that dependence on gravity alone requires quarter-root scaling for nonsimilar experiments. Furthermore, his results based upon similarity need not apply to nonsimilar experiments.

Usually, the scaling application of interest is not among similar experiments but, rather, among nonsimilar experiments. As demonstrated above with the choice of the mass, the energy, or the gravity set, similar experiments, using the same explosive type in a given soil, always scale as the cube root; but require variations of gravity. Scaling among such similar experiments is moot in that it only verifies that the controlling variables have been included in the analysis. Other scaling laws, e.g. quarter-root as shown above for the gravity set, can be achieved for similar experiments in the same soil only by varying the explosive and holding gravity constant and then only for strengthless materials. In any case, the results depend only on which other variables are held constant and not on the choice of π-groups.

Lastly, Chabai's statement (1965) that material properties must be scaled in order to preserve similarity is only true for constant gravity or for media which are dominantly strain-rate dependent or have inherent size properties.

The confusion surrounding the application of rules based upon similarity to nonsimilar experiments can perhaps be traced to misinterpretation of Sedov [1](1959), who was one of the first to recognize that gravity must be included in the analysis. His hypothesis was based upon using an incomplete set of variables: depth of burial h, soil density ρ_1, charge energy E and gravity g. This leads to a single π-group,

$$\pi_{SEDOV} = \frac{E}{\rho_1 \, gh^4} = \text{constant}, \tag{43}$$

and is equivalent to saying that all cratering experiments are similar.

Equation 43 appears to be the origin of the concept of quarter-root scaling, even though Sedov did recognize that his hypothesis was only an approximation which ignored "the effects of atmospheric pressure as well as the effect of internal elastic forces in identical materials." He did not, however, identify a dependence on Q_v or Q_e and his results are subsequently represented by cases 2.3 and 3.3 in Table 3. (Sedov's first Russian edition appeared in 1943, the third Russian edition in 1954 and the fourth Russion edition in 1956. Haskell (1955) appears to have independently identified the role of gravity in cratering and proposed a gravity modeling law which was simply $(L_1/L_2) = (W_1/W_2)^{1/4}$.)

In summary, similar experiments serve to verify understanding of the phenomenon. This includes the confirmation of a complete and consistent set of independent variables. In addition, similar experiments can be used to simulate other events. For example, by the use of a centrifuge small amounts of explosive can be used to simulate very large field events, as indicated by eq. 19c. However, it must be stressed that scaling rules based upon similarity need not apply to nonsimilar experiments, which is often assumed in the literature. Rather, as discussed, a set of nonsimilar experiments, such as the Nevada Test Site (NTS) series, will result in specific scaling rules based upon soil properties, explosive properties, and geometry of burst. The form of these scaling rules (e.g., the yield dependence) is determined by the analysis used to derive the relationships in Table 3.

SECTION 4

APPLICATION OF STATISTICAL-DIMENSIONAL ANALYSIS TO CRATER DATA

4-1 STATISTICAL MODEL DEVELOPMENT

In the previous section, various dimensionless groups and their restrictions on functional dependences were derived based upon plausible physical assumptions consistent with the experimental observation. The choice of which dimensionless groups are used in a particular application is theoretically arbitrary. The choice is ultimately based upon their utility and simplicity for explaining the data. A useful check on the dimensional analysis is to generate pi-groups based upon a statistical analysis of the original variables. A linear form consisting of a product of powers can be fitted to an n-dimensional array of n variables. The resulting powers can then be checked against the requirements of dimensional homogeneity providing a check that all variables have been included.

In recent laboratory scale experimental programs using a centrifuge, it was found that the mass set (eqs. 12a through 12d) provided a very good correlation for explosive crater formation at zero depth of burial.

To compare different soil types, the assumption of a Mohr-Coulomb strength theory was used to define the soil failure envelope for various stress states (Schmidt and Holsapple, 1979). The shear strengths at failure consist of two parts: a constant cohesion c and a contribution due to confining pressure p and the angle of internal friction ϕ.

$$S = c + p \tan \phi \tag{44}$$

Therefore, the generic strength Y, used to define π_3 in eq. 12c, is now interpreted to be the cohesion c

$$\pi_3 = \frac{c}{\delta Q_e} \tag{45}$$

The dimensionless quantity ϕ can be conveniently included as an independent π-group

$$\pi_5 = \tan\phi \tag{46}$$

For cratering at depth of burial d, an additional group is necessary

$$\pi_6 = d\ (\rho/W)^{1/3} \tag{47}$$

The complete set of π-groups to be used to statistically correlate the various data are given as follows:

$$\pi_1 = \frac{V\rho}{W} \tag{48a}$$

$$\pi_2 = \frac{g}{Q_e} \left(\frac{W}{\delta}\right)^{1/3} \tag{48b}$$

$$\pi_3 = \frac{c}{\delta Q_e} \tag{48c}$$

$$\pi_4 = \rho/\delta \tag{48d}$$

$$\pi_5 = \tan\phi \tag{48e}$$

$$\pi_6 = d\ \left(\frac{\rho}{W}\right)^{1/3} \tag{48f}$$

The ultimate goal is then to determine the dependence of the response variable π_1 as a function of the five independent variables

$$\pi_1 = H\ (\pi_2, \pi_3, \pi_4, \pi_5, \pi_6) \tag{49}$$

In principle, if there were in existence a multitude of cratering data for a common range of all the variables π_2 through π_6, a regression analysis on all five independent variables could be made. In the absence of such a multitude of data, restricted functional dependences based upon physical ideas or the limited experimental data are examined.

Consider, in particular, the special case of cratering at zero depth of burial (d.o.b.), $\pi_6 = 0$. For each combination of explosive type and soil medium, π_3, π_4 and π_6 are fixed. Thus, for each such combination there should be a single curve of π_1 versus π_2.

Figure 10 shows a series of such results, with each symbol type representing a fixed soil-explosive combination. There are a total of 91 points on the curve, including both laboratory and field shots. All explosives used in these shots were some type of chemical high explosive.

While each set of points for a given soil-explosive combination defines a particular trend, the entirety of points shows a very large spread. This indicates a need to account for a dependence on each of π_3, π_4 and π_6.

Now it seems plausible that the ratio of the stress magnitude generated by the explosive to the strength of the material is an important physical parameter. The stress or pressure due to the explosive at any point in the media is determined by the Chapman-Jouget (C-J) pressure P_{CJ} which is a characteristic of the explosive used. In terms of the properties defined previously, this C-J pressure is given by

$$P_{CJ} = 2(\gamma - 1)\delta Q_e \tag{50}$$

where γ is the perfect gas constant of the explosive products and has about the same value for all common explosives. Thus,

$$P_{CJ} \propto \delta Q_e \tag{51}$$

and the ratio of strength S to P_{CJ} is given by

$$\frac{S}{P_{CJ}} \propto \frac{c + p \tan\phi}{\delta Q_e} = \left(\frac{c}{\delta Q_e}\right) + \left(\frac{p}{\delta Q_e}\right)(\tan\phi) \tag{52}$$

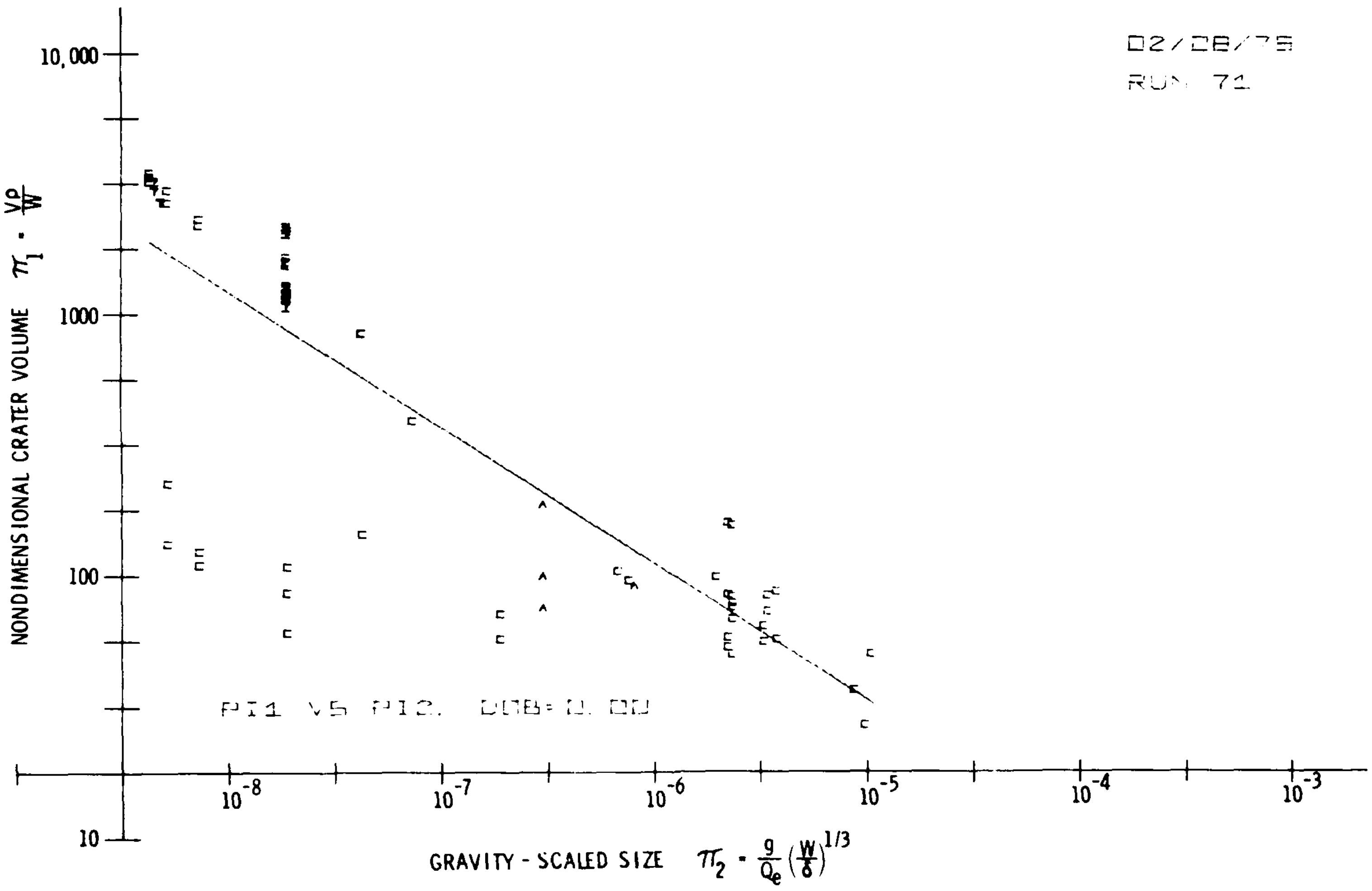


Figure 10. π_1 versus π_2, zero depth of burst data.

At this point, a measure of the confining pressure p is needed. It is commonly thought that the predominant cratering motions and material failures occur after the passage of the strong shock wave. The residual confining pressure is then the lithostatic pressure which at any depth h is equal to ρgh. Using this measure of confining pressure in eq. 52 gives

$$\frac{Y}{P_{CJ}} \propto \left(\frac{c}{\delta Q_e}\right) + \left(\frac{\rho gh}{\delta Q_e}\right) (\tan\phi) \tag{53}$$

The three terms in parentheses can each be written in terms of the previously defined π-groups (eqs. 48). The first is exactly π_3 and the last is exactly π_5. The other can be rewritten using eq. 11 as

$$\frac{\rho gh}{\delta Q_e} = \left(\frac{\rho g}{\delta Q_e}\right)\left(\frac{h}{r}\right)\left(\frac{3W}{4\pi\delta}\right)^{1/3} = \left[\left(\frac{3}{4\pi}\right)\left(\frac{\rho}{\delta}\right)\left(\frac{h}{r}\right)\right] \pi_2 \tag{54}$$

Now it is necessary to pick a characteristic dimension h, compared to the charge radius r. Clearly, an average or characteristic value for h should be a value between the charge radius r and the final crater depth d, which is typically on the order of a few times r. Consequently, for all experiments considered, the product of terms in square brackets in eq. 54 is on the order of unity and is henceforth dropped as inconsequential. With this simplification,

$$\frac{S}{P_{CJ}} = \pi_3 + \pi_2 \pi_6 \tag{55}$$

This combination of π-groups is useful in correlating cratering in materials with differing strengths. In this form, the gravity-size parameter π_2 occurs because of the dependence of strength on confining pressure proportional to size and gravity. In addition, the gravity-size parameter is important in its own right as a measure of the work done against gravity in excavating the crater. For a cohesionless material ($\pi_3 = 0$), only this latter dependence should remain. A new parameter formed in the simplest way as a linear sum

$$\bar{\pi}_2 = \pi_3 + \pi_2\pi_6 + k\pi_2 \tag{56}$$

$$= \pi_3 + \pi_2 (\pi_6 + k)$$

will reduce to a constant times π_2 when there is no cohesion. Data and auxiliary arguments indicate that for typical granular materials such as dry sand with tan ϕ on the order of 0.5 to 0.7, the term k should be smaller in comparison. A value of 0.1 has been found to work as well as any other for the present purposes. Consequently, a strength-gravity-size parameter is defined

$$\bar{\pi}_2 = \pi_3 + \pi_2 (\pi_6 + 0.1) \tag{57}$$

which will prove to be useful in correlating a range of data for various strength materials.

A plot of π_1 versus $\bar{\pi}_2$ is shown in Fig. 11. All of the data points now fall near a single curve, which is a straight line on this log-log plot. The combined strength-gravity-size parameter has worked exceptionally well as a parameter to correlate the large differences between various strength materials. The implications of the use of this $\bar{\pi}_2$ parameter are given elsewhere (Schmidt and Holsapple, 1979) and will not be duplicated here.

The Mohr-Coulomb-failure envelopes were constructed from static triaxial test data. Values for cohesion and angle of internal friction are based directly upon this measured data where available. Some of the data shown in Figs. 10 and 11 are from the 1-G laboratory-scale experiments performed by Piekutowski (1974, 1975, 1978). These shots were all in dry sand at various densities. For these, no direct measurements of soil-strength properties for the various densities were made. It is apparent that those at lower bulk density have less angle of internal friction. The approach taken was to determine what variation of ϕ with ρ would best correlate the shots, using the theory given above, to see if the results were reasonable. Fig. 12 shows the final choice of ϕ versus ρ for these dry-sand events. The results seem plausible.

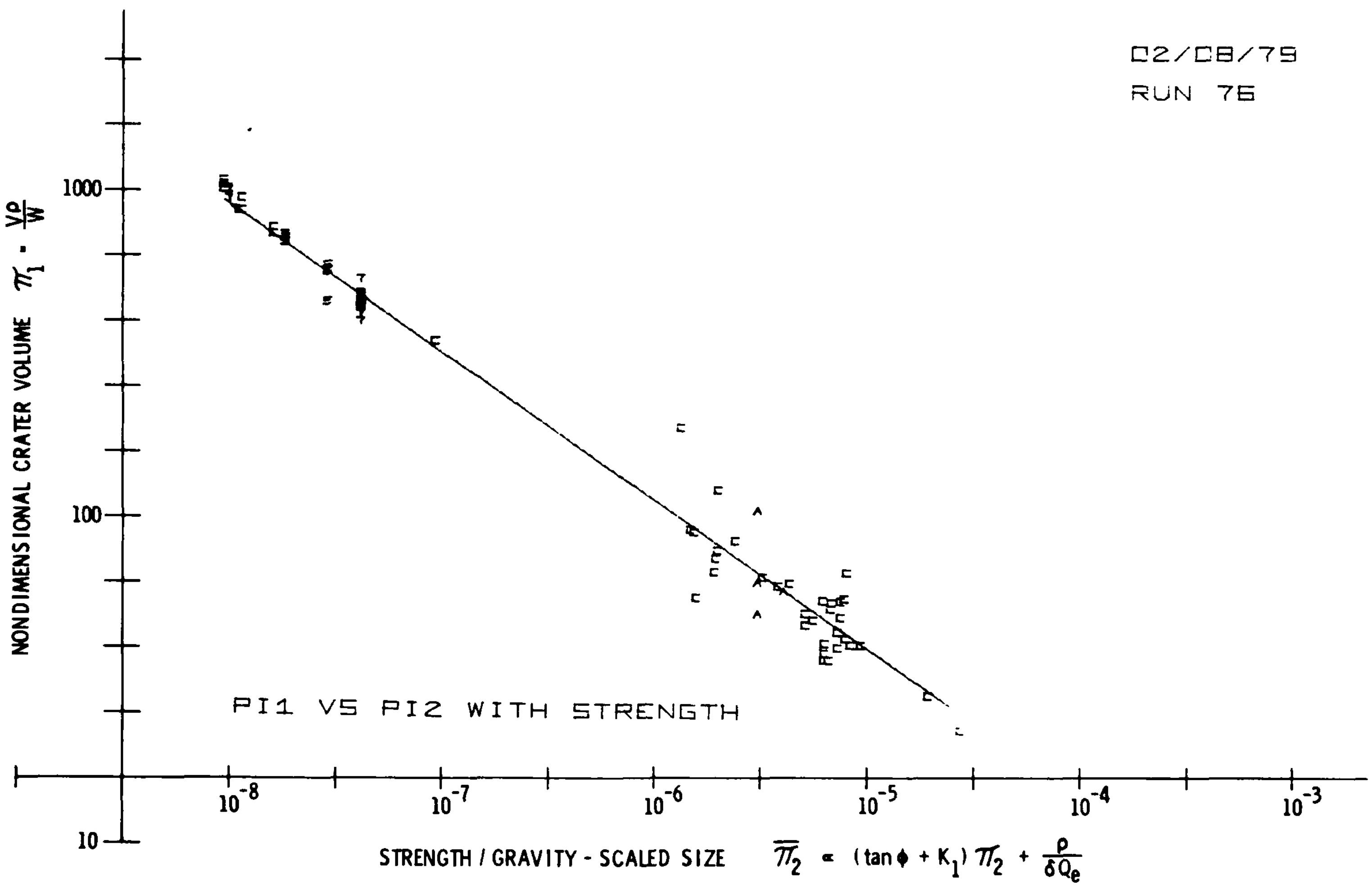


Figure 11. π_1 versus $\overline{\pi}_2$, zero depth of burst data.

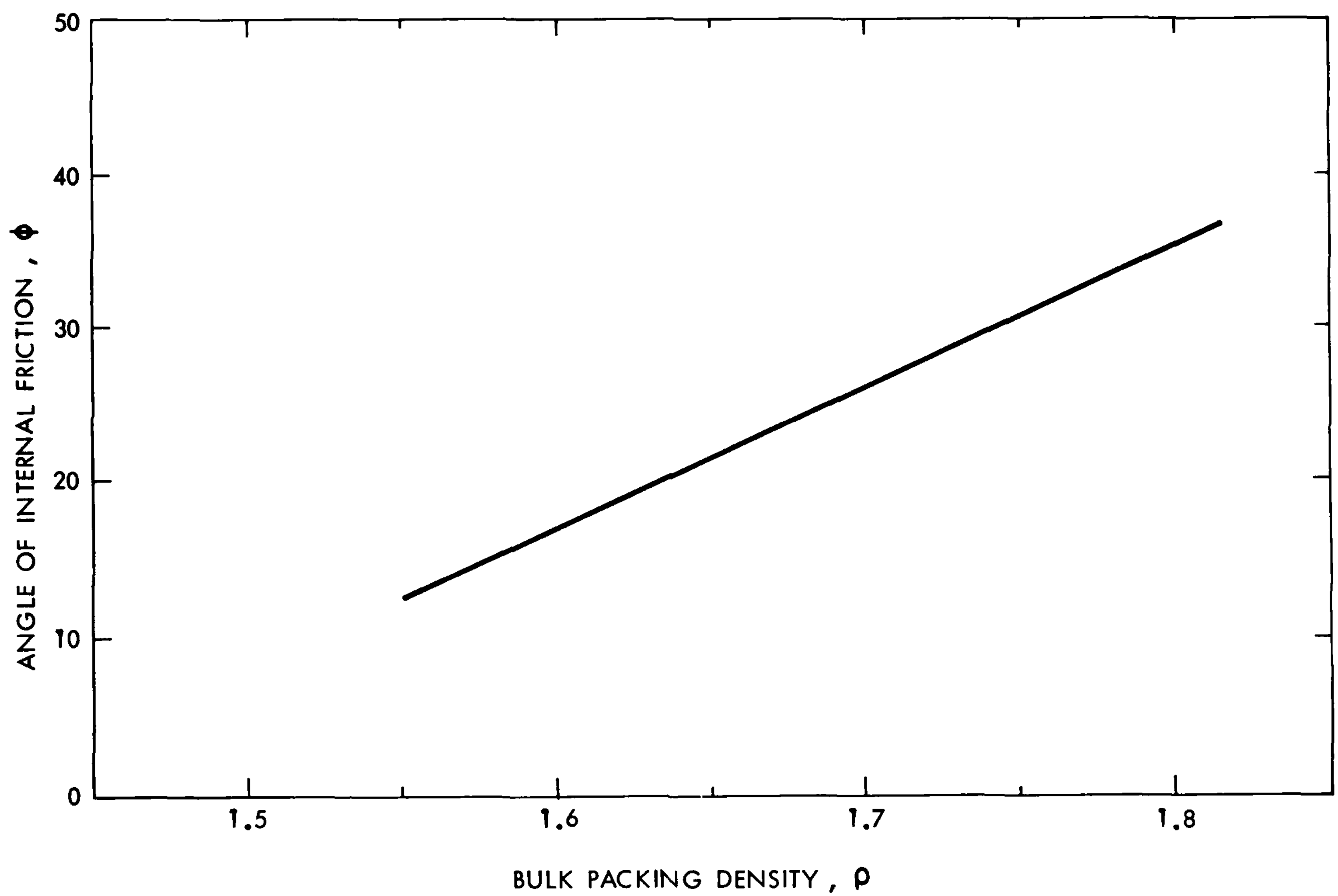


Fig. 12. Variation of friction angle with packing density for dry Ottawa Flintshot sand

Once all of the zero-d.o.b. shots were correlated with the effects of varying media and explosive properties, it was possible to consider the extension to all values of the d.o.b. π-group π_6. For the zero π_6 shots, the final correlation was fit by the curve

$$\pi_1 \, \bar{\pi}_2^{\alpha} = \text{constant} \tag{58}$$

where the best choice for α is about 0.47. The parameter $\bar{\pi}_2$ incorporates all effects of π_2, π_3, π_4, π_5, and π_6, at least for the zero d.o.b. shots ($\pi_6 = 0$).

If on the plot of π_1 versus $\bar{\pi}_2$, the events with various π_6 are plotted while there is increasing scatter, the various π_6 lines are essentially parallel to the $\pi_6 = 0$ line on the log-log paper. Consequently, the correction for non-zero π_6 is assumed to be independent of $\bar{\pi}_2$, and a total functional form is given by

$$\pi_1 \, \bar{\pi}_2^{\alpha} = C(\pi_6) \tag{59}$$

This indicates the usefulness of a plot of $\pi_1 \, \bar{\pi}_2^{\alpha}$ versus π_6 for all available shots.

Such a curve is shown in Fig. 13. It is seen that all 226 points, most of which overlay to the extent that they cannot be distinguished from one another, fall very close to the curve shown. Only at large d.o.b. is there much scatter. This is scatter from field shots in desert alluvium that were presumably reproducibility tests for identical shot configurations. No significantly better fit to the results of these 226 points can be found using this model. (Note that Fig. 11 is a cross plot of this curve at $\pi_6 = 0$.)

To characterize the behavior near zero d.o.b., a range of π_6 from -2.0 to +2.0 is shown separately on an expanded scale in Fig. 14.

Alternative results using the same 226 points which should be compared with Fig. 14 are given in Figs. 15 and 16. Figure 15 is a plot of π_1 versus π_6 and therefore has no correction for size, strength or gravity. These correlation parameters are essentially those used by Dillon (1971) and others

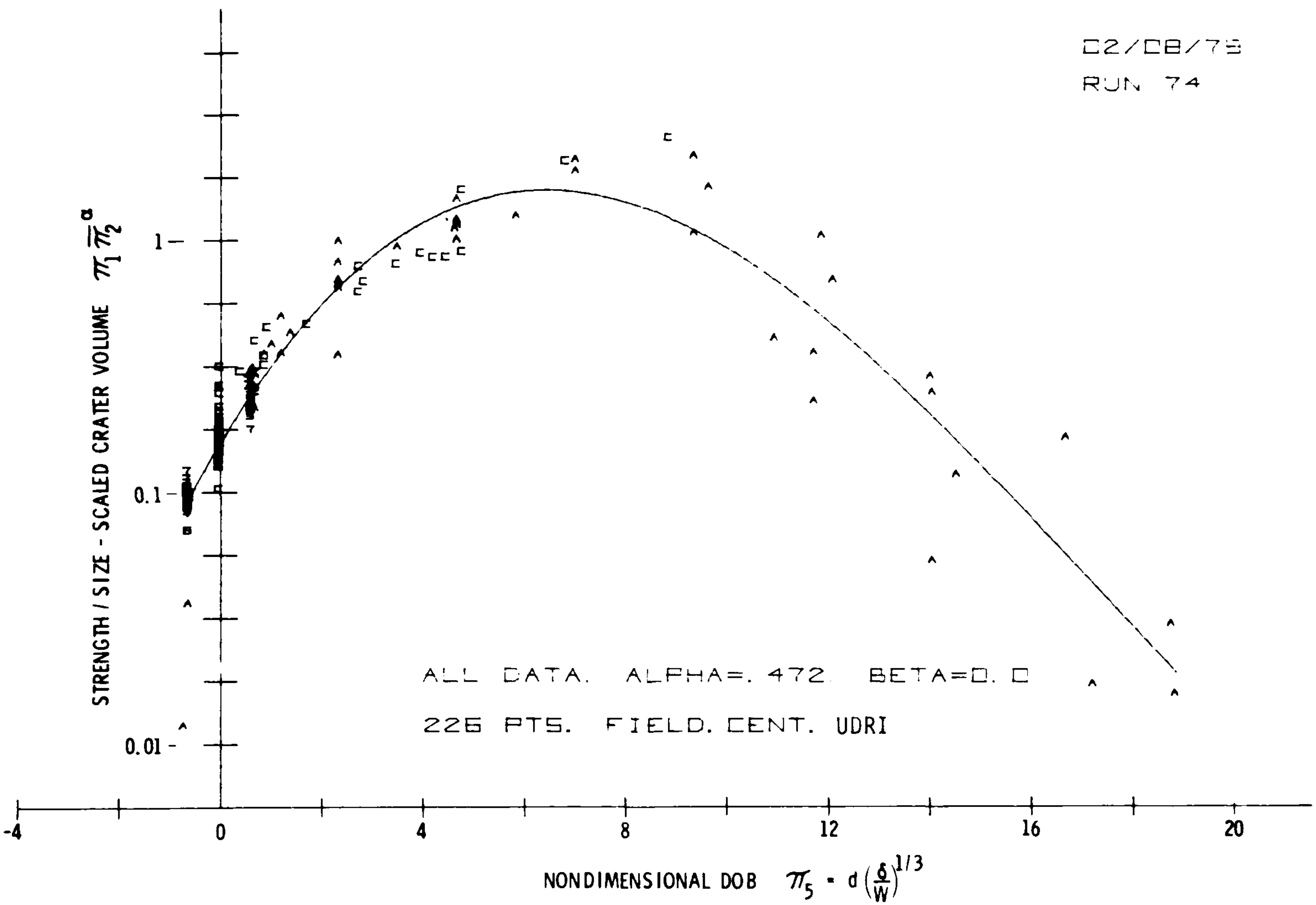


Figure 13. $\pi_1 \bar{\pi}_2^{\alpha}$ versus π_5. All data.

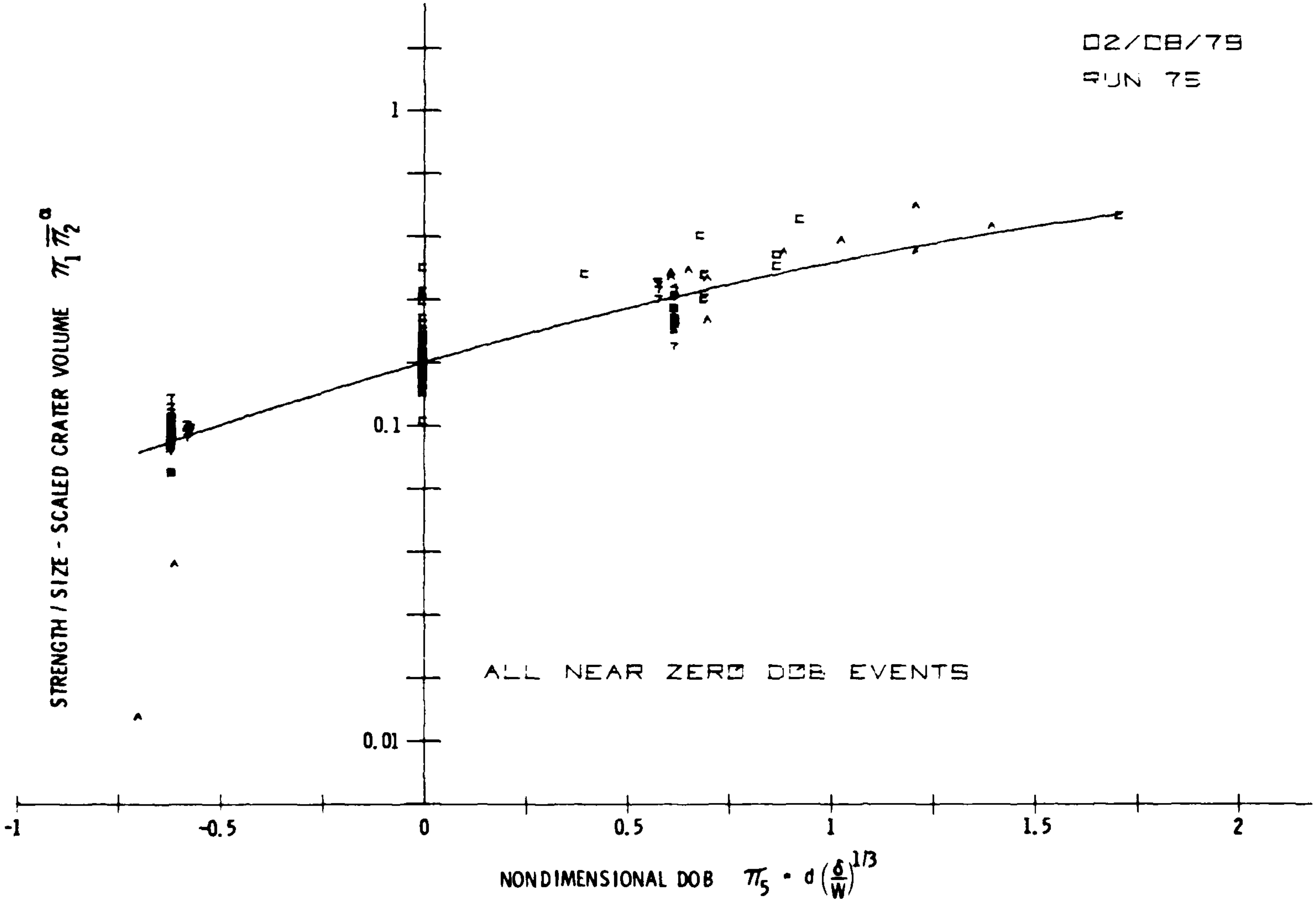


Figure 14. $\pi_1 \overline{\pi}_2^{\alpha}$ versus π_5. Near surface burst events.

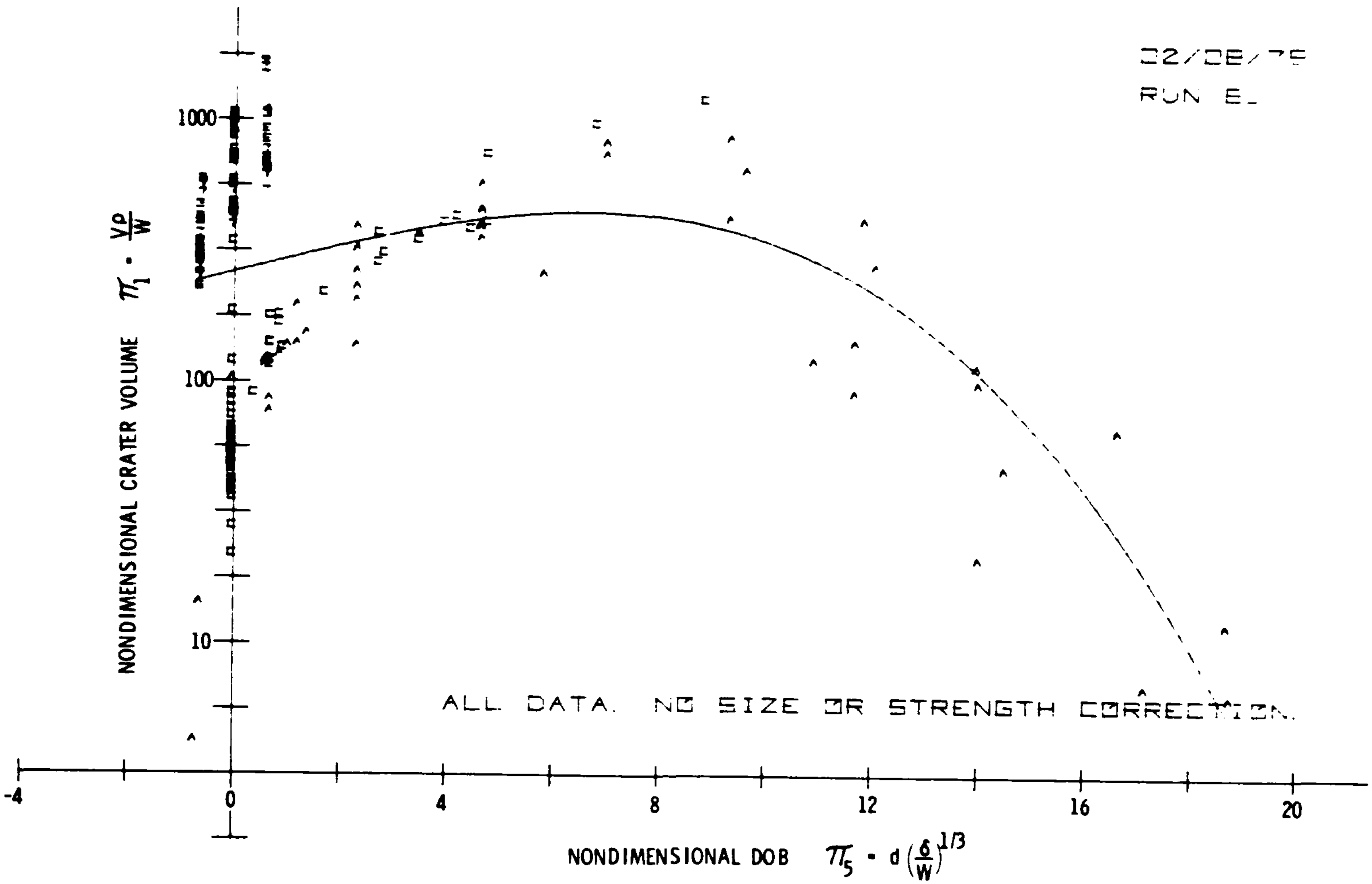


Figure 15. π_1 versus π_5. No strength or size correction.

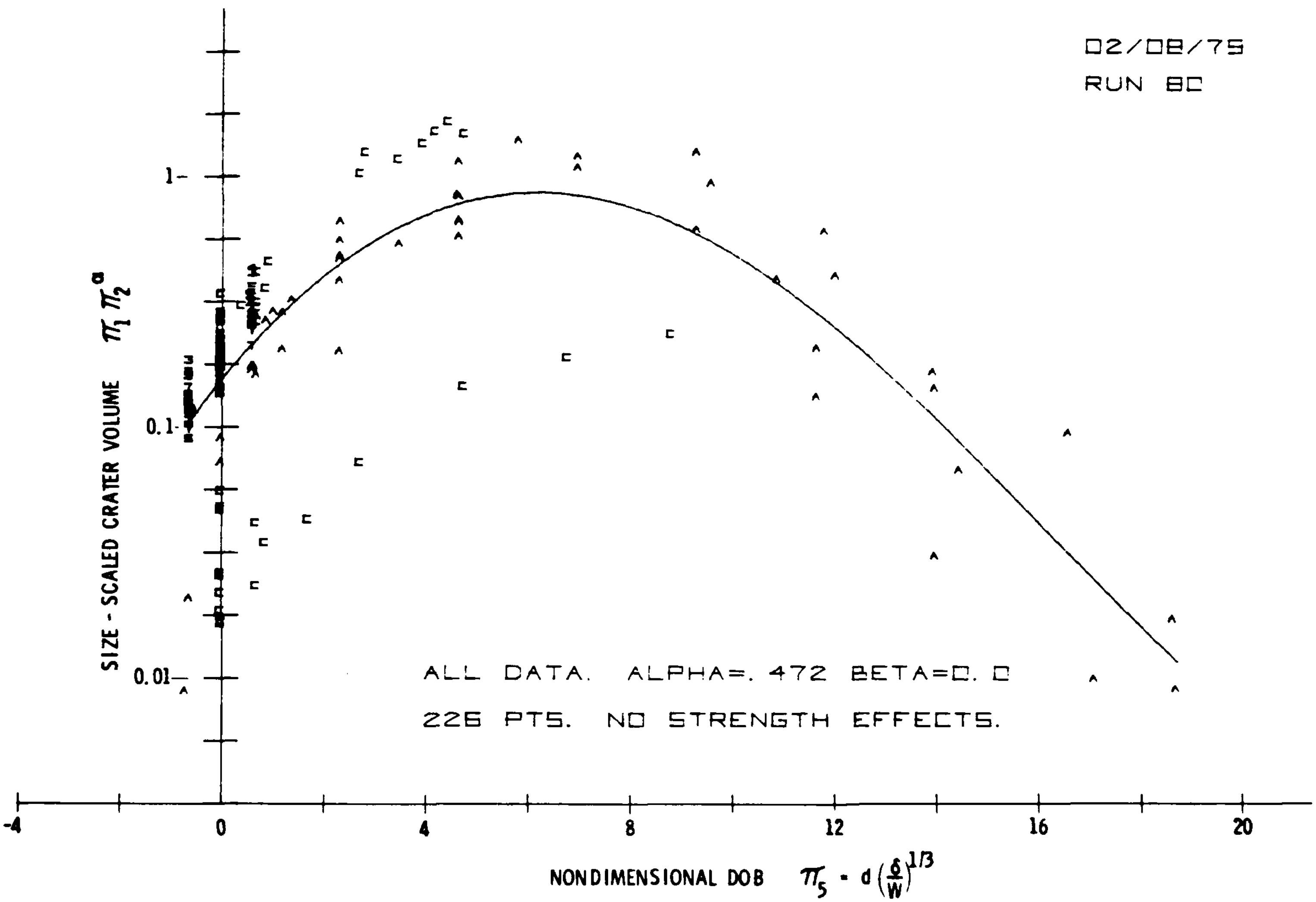


Figure 16. $\pi_1 \pi_2^{\alpha}$ versus π_5. No strength effects.

with little success. Figure 16 has the size correction, and is thus a plot of $\pi_1 \; \pi_2^{\alpha}$ versus π_5, but there is no strength correction. These two figures demonstrate that there is significant reduction of scatter by incorporating the strength-gravity- size parameter discussed above.

4-2 DETAILS OF STATISTICAL-DIMENSIONAL ANALYSIS

The above arguments indicate that a reasonable functional form for a model to statistically correlate the data is

$$\pi_1 \; \bar{\pi}_2^{\alpha} = J \; (\pi_6, \; \bar{\pi}_2^{\beta}) \tag{60}$$

Furthermore, trial and error showed that the data are well modeled by taking the log of the left-hand side to be a polynomial in the parameter $\pi_6 \, \bar{\pi}_2^{\beta}$ as

$$\log \; (\pi_1 \; \bar{\pi}_2^{\alpha}) = \sum_{i=0}^{n} a_i \; (\pi_6 \; \bar{\pi}_2^{\beta})^i \tag{61}$$

The fit shown in Fig. 13 was based on a 4th order polynomial that reduced to a lower order polynomial as shown. Higher orders up to 9th order were considered but discarded at this stage for simplicity since they consistently provided no additional terms. This will be discussed below. Equation 61 may be rewritten as.

$$\log \pi_1 = -\alpha \log \bar{\pi}_2 + \sum_{i=0}^{n} a_i \; (\pi_6 \; \bar{\pi}_2^{\beta})^i \tag{62}$$

Note that the equation is linear in α and a_i and nonlinear in β. An alternative was to try a nonlinear fit using these seven coefficients. However, a nonlinear optimization rarely works well with that many parameters. For this reason an iterative procedure was used. First, a nonlinear fit was used for the power

coefficients α and β to obtain a value which minimizes the sum of squares. A linear regression was then run on the a_i. After finding the a_i which further decrease the sum of squares, the nonlinear optimization for the α and the β was rerun. Since at each step this will decrease the sum of squares and since the sum of squares is non-negative, after a finite number of tries one must be within a reasonable distance of a local minimum. This was non-productive, however, due to a paucity of depth-of-burial data at large $\bar{\pi}_2$. Consequently, β was set to zero since it proved to be a totally insensitive parameter for this limited data set. This may not be the general case and should optimum excavation again be of interest, more data should be collected. This could be done inexpensively by using a centrifuge.

The model was simplified as much as possible using what is called a stepwise-multiple-regression program. First it was assumed that the fourth-order polynomial has only a constant term. At each step an additional coefficient was added if the coefficient was statistically significant when added to the prediction equation. When no more terms can an be added, the program terminates. In practice it gave a simpler model than the whole ninth-order polynomial, usually a cubic.

This program is illustrated by the following discussion on the output from the linear and nonlinear programs used for this study. The discussion will proceed referencing the specific printed computer output (Appendix A) which is annotated with large capital letters near the right margin. These letters correspond to the following paragraphs so labeled.

A. This is the start of the computer run. There are a total of 226 observations, and the total number of variables is 5 of which the 5th variable is the dependent or response variable; that is, the variable to be predicted. Variable 6 would consist of a dummy code always taking the value 1 which would be the constant term. The constant term is always in the regression. At each step a new variable will be allowed to enter the regression only if the added amount of prediction is statistically significant at the 0.05 level. If after a certain number of variables are added to the predictive equation and a variable already in the regression turns out to be no longer statistically significant at the 0.15 level, that variable will be dropped from the equation. That is, the

variable dropped has a significant level less than 0.15 for adding to the regression equation.

B. The output then gives the means, standard deviations and the matrix of correlation coefficients between variables. Recall that the square of the correlation coefficient between two variables tells how much of the variability in one variable can be explained linearly with the other variable. The correlation coefficient has a plus sign if the variables increase together or a minus sign if they tend to go in the opposite direction.

C. The only variable added here is the variable 6, or the constant term. Thus, there is no prediction beyond prediction with a constant. Note that in this case the R^2 value or the square of the multiple correlation coefficient is zero. The prediction is as good as it can be using a constant. At each step the variables in the equation will be printed out along with the value of the coefficient a_i. Note in this case, predicting only with a constant, the coefficient is -0.693. This, of course, is the mean of the response variable, as can be checked above the mean of variable 5 is -0.6935.

D. At this point partial-correlation coefficients are computed for the variables not in this equation. The following describes the meaning of the partial correlation coefficient. At each step in the stepwise regression, some variables, even if only the constant term, will already have been added. A partial correlation is the correlation between the dependent or response variable and the variable noted after the effect of the other variables in the prediction has already been taken out. To compute a partial correlation one subtracts the best prediction for both the dependent variable and the variable considered in terms of the variables already in the predictive equation. For example, variable 3 has a partial correlation with the dependent variable -0.163. In other words, after subtracting out the best prediction for the dependent variable and variable 3 in terms of a constant predictor, (i.e., their means) the amount of variability left in the dependent variable that can be explained by variable 3 is a square of this number. The minus sign shows us that the dependent variable tends to decrease as variable 3 increases.

E. At each step, the variable that adds the most to the prediction (provided it is statistically significant) is chosen to enter the equation. In this case, that variable is variable 1. Note, comparing C and E that the standard deviaton of the residuals (that is, the square root of the sum of squares of the residuals divided by N minus the number of predictors being used) has decreased. In addition, the fraction of variability explained has increased from 0.00 to 0.08.

Following the "multiple-correlation coefficient squared" is a table called an analysis of variance (ANOVA) table. The total sum of squares is the sum of the squares of the observed, dependent variable minus the mean. Each stage can be broken up into two sum-of-squares groups: the sum of squares of residuals, discussed earlier, and the rest of the sum of squares which is the part explained by the regression equation. If the errors are normally distributed, these sums of squares may be used to calculate a statistic which has a distribution called an F distribution. To get the F distribution, each sum of squares is divided by its degrees of freedom, or DF. This is a term indicating how many dimensions of the data points, or how many data points, effectively contribute to the variability expressed in the sum of squares. Dividing the sum of squares by the degrees of freedom gives a mean square. The ratio of the two mean squares is the F statistic. The first number of the degrees of freedom is the numerator degrees-of-freedom and the second number the denominator degrees-of-freedom. The statistical significance of the overall prediction can be found in the F distribution tables. Following this, we once again have the coefficients a_i, using the fit with a constant variable and variable 1. To the right of that are the standard errors of the estimated coefficients.

F. Proceeding to the next line, a new set of partial correlation coefficients is given. Note the great change in going from D to F. The correlation of variable 2 that was quite small before (in the range of -0.34) is now quite large. This occurs because variable 1 contained most of the predictive information compared to the remaining variables. Once variable 1 is in the equation, the remaining variables are about equally effective in explaining the remaining variability in the dependent variable.

G. The program proceeds step by step increasing the value of multiple-correlation coefficient squared. Occasionally, when a variable is added, a partial correlation coefficient increases because the variables left may not have predicted as well before, but, when added to the predictive ability of the variable just included, they help predict more of the remaining variability.

H. This is the last step, which added variable 3. The partial-correlation coefficients printed below G indicate there are no values large enough to be statistically significant. At this point, the program again writes out the coefficients and the dispersion matrix of the coefficients. The dispersion matrix gives the estimated variances and covariances for the errors in the coefficients. This gives some idea of the precision with which the coefficients are estimated. It also helps to establish confidence intervals for various quantities estimated in terms of the parameters a_i.

I. Finally the success of the prediction can be examined, as well as some idea of the appropriateness of the normal model, by looking at the observed and predicted values and their residual values, which, of course, are the difference between the two. The residuals will be approximately normally distributed if the model is correct. The far right-hand column gives the standardized residuals, which are the residuals divided by their standard deviation. Values greater than 2.0 or, at most, 3.0 in absolute value will not tend to occur if the sample is normal. Such values indicate that the data has outliers which should be examined.

J. This is an example of the output using the nonlinear-least-squares program used in this study. The program proceeds iteratively to try to find a minimum for the residual sum of squares. As can be seen from the two lines designated "residual sum of squares," the estimated residual sum of squares has decreased a little. The values for α and β have gone from starting points of 0.5 and 0.167 to 0.499 and 0.1667. The sum of squares changes very little because of the accuracy of the initial starting point.

4-3 ESTIMATING FUNCTIONS FROM THE ESTIMATED MODEL INCLUDING A MEASURE OF THE ACCURACY OF PREDICTION

Suppose that we wish to compute some function of our parameter values, called θ_1 up to θ_p at a datapoint $\underline{x}$. Let the function be called y as in eq. 63.

$$y = f(\theta_1, \theta_2, \ldots, \theta_p, \underline{x}) \tag{63}$$

We would like to estimate the value at the true parameter value. Let these be θ_1° through θ_p°. We want to estimate y_0 as given in eq. 64.

$$y_0 = f(\theta_1^\circ, \theta_2^\circ, \ldots, \theta_p^\circ, \underline{x}) \tag{64}$$

The estimated parameter values, θ_1 through θ_p, will not be the true values, but, for a large sample size, they will be close. We may expand the function f about the true parameter values in a Taylor series, taking the first order terms. This leads to the approximate equality of eq. 65 below.

$$y - y_0 \approx \sum_{i=1}^{p} (\theta_i - \theta_i^\circ) \frac{\partial f}{\partial \theta_i} \tag{65}$$

If we square both sides of this equation and take the expected value we are led to eq. 66. Since the method of maximum likelihood in this case gives estimates whose expected value is the true value asymptotically, eq. 67 follows.

$$E[(y - y_0)^2] \approx \sum_{i=1}^{p} \sum_{j=1}^{p} \frac{\partial f}{\partial \theta_i} \frac{\partial f}{\partial \theta_j} E[(\theta_i - \theta_i^\circ)(\theta_j - \theta_j^\circ)] \tag{66}$$

$$\text{Std. Dev. } y \approx \sqrt{\sum_{i=1}^{p} \sum_{j=1}^{p} \frac{\partial f}{\partial \theta_i} \frac{\partial f}{\partial \theta_j} \text{Covariance } (\theta_i, \theta_j)} \tag{67}$$

Thus, we see that we may get some idea of the variability in the estimate y from knowing the partial derivatives of the function we are estimating and the covariance of the parameters estimated.

This may be usefully employed to estimate the variability of various predicted quantities.

SECTION 5
RESULTS

From dimensional analysis alone, limits were derived on functional dependences for crater scaling laws. The general results shown in Table 3 are applicable to the assumptions put forth in Section 3-4. These conclusions reinforce the hypothesis that scaling should be bounded by quarter-root and cube-root laws. More importantly, independence of certain variables is necessary to achieve these limits. For example, as shown by case 2.4, quarter-root scaling can only be achieved if the crater volume is independent of both material strength and source specific energy. This latter independence has not been demonstrated.

Seven generalizations leading to the results in Table 3 follow from the analysis and are given in the text following eq. 30. Modifications attributable to the inclusion of a more general class of material behavior are also given and affect only three of the previous conclusions.

These results are general and can be used directly to approximate limiting behavior for different regimes of charge size and soil type. More importantly, for this study, they provide the framework for selecting a statistical model. By virtue of the dimensional analysis alone, the number of independent variables has been reduced to a set of independent π-groups three less in number. This simplification allows fitting a statistical model in two or, at most, three dimensions permitting straight-forward model fitting using graphical representation for further visual evaluation.

Figures 10 through 16 show how the role of the different governing variables can be determined. This leads to combinations of π-groups which themselves are also new, more complex π-groups. By successive combination of appropriate π-groups, using statistical model fitting at each step, the total dependence upon all the independent physical variables can be included. In the present example, for the apparent volume dependence, the procedure went as follows.

Non-dimensional volume π_1 in a given material for constant d.o.b. was observed to be a simple function of a gravity-scaled-size parameter π_2. The variability due to different material types was then correlated using a strength-gravity-parameter $\bar{\pi}_2$, again at constant d.o.b. as shown in Fig 11.

Achieving this for constant d.o.b. gave a useful form for a scaled volume $\pi_1 \; \bar{\pi}_2^{\alpha}$, which was then correlated with a nondimensional d.o.b. as shown in Fig. 13. A general nonlinear form given by eq. 61 was tried first. However, with the large proportion of the near-surface data base used in this study, no significant improvement in the fit could be found, and β was set to zero, resulting in the final fit shown in Figs. 13 and 14. This zero correlation for β was due to a paucity of large-yield-explosive data.

Before this more elaborate non-linear d.o.b. correlation ($\beta \neq 0$) is tried further, the compendium of data must be further expanded to include all available large-yield data, especially at depth. This could include the nuclear data which was deliberately omitted in this pilot study to avoid additional variability due to source characteristics. Once a reliable model for cratering with conventional explosives is determined, it can be used to determine appropriate nuclear source characteristics. It is better to formulate statistical models from high-quality, well-characterized data. This helps to identify trends due to material strength, size, gravity, etc., which can then be eliminated from the total variability. Then the nuclear data can be added leaving only the source variability to be characterized.

Foremost within the scope of this pilot program, the objective was to demonstrate the utility of a combined statistical-dimensional analysis technique using a well-defined, high-quality data base. In many cases, the experimental test conditions and soil-site surveys are not properly documented or require extensive search to recover some of the older documents. This is a serious problem for any statistical correlation of cratering data or for free-field environment. The scope of this program did not allow a totally comprehensive search, although a compendium of more than 1500 cratering events was cataloged and put on computer cards. A summary of this compendium is given in Appendix B. From this master compendium, selected data were chosen and added in consecutive groups to the working compendium. This permitted a controlled expansion of the

data base, allowing a careful second screening of less obvious inconsistencies, due to source or soil descriptions in particular, at each step. The working compendium consisted of 226 events at the end of the study. Another advantage of working with a characteristic, but high quality subset of data is efficiency of computer cost and turn-around while sorting out fundamental forms for the statistical model.

The SDA methodology is an iterative approach utilizing simple statistical models to determine interdependences of various π-groups. From these, more complex π-groups are developed and, again, statistically correlated. The advantages of this method are manyfold. The dimensional analysis not only reduces the number of independent variables, but provides nondimensional forms allowing direct comparison among grossly heterogenous data. This includes very small-scale laboratory and field data, centrifuge data, large-scale field events, and, ultimately nuclear data. Source-type variations can also be compared on a common basis allowing the utilization of impact data as well as high explosives to further formulate nuclear parameters (Schmidt and Holsapple, 1978b).

At all stages, *physical* models can be used to formulate the statistical model. This avoids the common problem of using a high-order polynominal in each physical variable--all represented simultaneously in an n-dimensional space. In addition to precluding any simple graphical representation, confidence in prediction is reduced due to excessive dependence on a large number of parametric coefficients. In contrast, the SDA hybrid approach optimizes the functional form of the scaling relationships, thus, minimizing the number of free coefficients. This is done through consecutive recombination of variables using arguments of dimensional analysis prior to each statistical-regression analysis.

SECTION 6
RECOMMENDATIONS

The SDA hybrid methodology can be applied to any physical problem. The example developed in this pilot study is improved correlation of crater volume. For cratering and associated phenomena, the study should be extended to examine and to improve correlation for crater radius and for crater depth. Preliminary observations indicate that these two response variables are much more sensitive to geological site and material properties than is the crater volume. Statistical techniques beyond the scope of the present study that can be used to improve and extend the utility of this method include the following.

6-1.1 More Predictive Variables

As more π-groups are considered, it becomes cumbersome to look at the data. The software should be upgraded to allow for a variety of plots, such as residuals and predicted values versus various quantities. This could be accomplished using on-line-interactive graphics now available. Discussions of various diagnostic plots are given by Fisher and vanBelle (1979). In addition to more plotting methods, there are analytical means to test for outliers in the residuals. These should be implemented for ease in locating data points that do not appear to fit the model well. The normality assumption crucial to setting up estimates of variability, such as confidence intervals, may also be examined by test statistics and by looking at normal probability plots of the residuals.

6-1.2 Suitability Of Other Models

There are numerous other analytical models other than the models tested to date that might be tried as fits to these data. As mentioned above, the fewer the parameters in the model, the more confidence one has that the extrapolation to other values will work. It is difficult to say precisely what can be done in this area because model fitting is as much an art as a science and therefore is best implemented through experience based upon physical models.

6-1.3 Other Measures For Goodness Of Fit

A particular virtue of the current work is the emphasis upon using similar experiments to relate small-scale laboratory data, high-gravity-centrifuge data, and large-scale field data. This is done in terms of the π-groups and has proved an efficient and valuable method of explaining variability in the data. Nevertheless, for certain purposes one may have an idea of the loss associated with prediction in terms of data in original units rather than in nondimensonal groups. The predictions could be transformed back to the "original" spaces for examination. Such plots would be useful in assessing the potential gains and possible need for additional centrifuge data.

While variability methods are precise if the residuals are normally distributed and all of the model assumptions hold, one is usually not willing to assume that the model holds precisely. Variability and accuracy of prediction can be better estimated by using the so-called jackknife method or the subsampling method. In the former, one data point at a time is held out and the estimates for the model are found based upon the remaining data. The accuracy of the prediction when using the point left out is then assessed. (There is a gain in looking at the accuracy of the model for each case, because the point left out does not influence the curve fit and would be expected to be more in error.) The subsampling method, which takes much less computer time than the jackknife method (which reruns the problem a number of times equal to the number of datapoints), selects a subsample, such as leaving out a third of the data points at random. The remaining points are then used to get a fit to the model and the accuracy of prediction for the points left out is observed. These approaches can be automated in the software.

It is particularly easy to do this for linear models where the computations are not prohibitive. For nonlinear models, considerable computer time is needed to implement such approaches extensively.

Finally, for some purpose one may be willing to assign a loss function to inaccuracies in prediction. The loss function may or may not be the least-squares model. If it does not correspond to least squares one may look for

predictive methods more accurate for the loss function considered. In this case the statistical background is less well developed and there would be a need to work on some methodological developments.

6-2 APPLICATION TO FREE-FIELD ENVIRONMENTS

In addition to crater environment, an obvious and important extension of the SDA methodology is the application to free-field ground motions. The practical feasibility for horizontal displacement has been demonstrated by Holsapple et al. (1978) using simple functional dependences given by Cooper and Sauer (1977). Vertical displacement, velocity, acceleration, and stress as a function of range, geology, and source are not all presently well correlated. The advantage of the SDA approach is that it quantifies the statistical variability common to field geologies for all stages in the model development. This isolates the physical model from spurious data and allows a stepwise construction of a complex statistical model which never exceeds two- or three-dimensional function space. As shown in the above model for crater volume, a simple third-order polynomial with five free coefficients provided a very good fit for 226 data points. More importantly, the simpler the final model, the more reliable are predictions based upon it. It also provides a high degree of confidence in estimating the uncertainty due to unexplained variability of the data.

This method is a systematic, stepwise approach incorporating physical, mathematical and statistical techniques to the development of scaling laws in general. By iterating between the physical model and the statistical model using dimensional analysis, one can always see the effect of modifications incorporating additional variables or different mathematical forms.

SECTION 7
REFERENCES

Baker, W. E., P. S. Westine and F. T. Dodge (1973) Similarity methods in engineering dynamics: Theory and practice of scale modeling, Hayden Book Company, Inc., New Jersey.

Buckingham E. (1914) On physically similar systems; Illustrations of the use of dimensional equations, Phys. Rev. IV, 4, pp. 345-376.

Burton, D. E., C. M. Snell, and J. B. Bryan (1975) Computer design of high-explosive experiments to simulate subsurface nuclear detonations, Nuclear Technology vol. 26, pp. 65-87.

Chabai, A. J. (1965) On scaling dimensions of craters produced by buried explosives, J. Geophys. Res., vol. 70, pp. 5075-5098.

Chabai, A. J. (1967) Dimensional analysis and modeling of cratering explosions, Proceedings of Symposium on Nuclear Craters and Ejection, Aerospace Report No. TR-1001 (S2855-40)-4 vol. 1, Section C.

Chabai, A. J. (1977) Influence of gravitational fields and atmospheric pressures on scaling of explosion craters, in Impact and Explosion Cratering, edited by D. J. Roddy, R. O. Pepin, and R. B. Merrill, pp. 1191-1214, Pergamon Press.

Cooper, H.F. (1977) A summary of explosion cratering phenomena relevant to meteor impact events, in Impact and Explosion Cratering, edited by D. J. Roddy, R. O. Pepin, and R. B. Merrill, pp. 11-14, Pergamon Press, NY.

Crowley, B. K. (1970) Scaling criteria for rock dynamic experiments Symposium on Engineering with Nuclear Explosives, vol. 1, pp. 545-559, American Nuclear Society and U. S. Atomic Energy Commision, Las Vegas.

Dillon L. A. (1971) The influence of soil and rock properties on the dimensions of explosion produced craters, AFWL-TR-71-144, Air Force Weapons Laboratory, Albuquerque, NM.

Divoky, D. (1966) The equivalence of mass and energy scaling of crater dimensions: comments on a paper by A. J. Chabai, J. Geophys. Res., vol. 71, p. 2691.

Fisher L. D., and G. vanBelle (1979) Biostatistics: A methodology for the biomedical sciences, Duxbury Press, Boston (unpublished).

Gault, D. E., and J. A. Wedekind (1977) Experimental hypervelocity impact into quartz sand--II, Effects of gravitational acceleration, in Impact and Explosion Cratering, edited by D. J. Roddy, R. O. Pepin and R. B. Merrill, pp. 1231-1260, Pergamon Press, New York.

Haskell, N. A. (1955) Some considerations on the modeling of crater phenomena in earth, Air Force Surv. Geophys., vol. 67, TN-55-204, Air Force Cambridge Research Center, Bedford. MA.

Herr, R. W. (1971) Effects of the atmospheric-lithostatic pressure ratio on explosive craters in dry soil, NASA TR R-366, National Aeronautics and Space Administration, Washington DC.

Holsapple, K. A. (1979a) Material strength and explosive property effects in cratering and ground shock, Proceedings of Sixth International Symposium on Military Applications of Blast Simulation, Cahors, France.

Holsapple, K. A. (1979b) Impact cratering experiments with ambient atmospheric pressure (abstract). EOS (Trans. Am. Geophysics. U.) 60, p. 871.

Holsapple, K. A., and R. M. Schmidt (1979) A material-strength model for apparent crater volume. Proc. Lunar and Planetary Science Conference 10th, pp. 2757-2777, Pergamon Press, NY.

Holsapple K. A., R. M. Schmidt and R. L. Dyrdahl (1978) Gravity-scaling methods applied to crater-induced ground motions and effects. Proc. of Nuclear Blast and Shock Simulation Symp., Defense Nuclear Agency-SPSS, Nov. 28-30, 1978, San Diego, CA.

Killian, B. G., and L. S. Germain (1977) Scaling of cratering experiments--An analytical and heuristic approach to the phenomenology, in Impact and Explosion Cratering, edited by D. J. Roddy, R. O. Pepin, and R. B. Merrill, pp. 1165-1190, Pergamon Press, New York.

Knowles, C. P., and H. L. Brode (1977) The theory of cratering phenomena, an overview, in Impact and Explosion Cratering, edited by D. J. Roddy, R. O. Pepin, and R. B. Merrill, pp. 869-895, Pergamon Press, N.Y.

Maxwell D., J. Reaugh, and B. Gerber (1973) JOHNIE BOY crater calculations, Report DNA 3048F, Defense Nuclear Agency, Washington, D.C.

Orphal, D. L. (1977) Calculations of explosive cratering-I: The shallow-buried nuclear detonation JOHNIE BOY, in Impact and Explosion Cratering, edited by D. J. Roddy, R. O. Pepin, and R. B. Merrill, pp. 897-906, Pergamon Press, N.Y.

Schmidt R. M. (1977) A centrifuge cratering experiment: Development of a gravity-scaled yield parameter. Impact and Explosion Cratering (D. J. Roddy, R. O. Pepin and R. B. Merrill, eds.) Pergamon Press, New York.

Schmidt, R. M. (1980) Meteor crater--Implications of centrifuge scaling (abstract). In Lunar and Planetary Science XI, pp. 984-986, Lunar and Planetary Institute, Houston, TX.

Schmidt, R. M., and K. A. Holsapple (1978a) Centrifuge crater scaling experiments I: Dry granular soils, Report DNA 4568F, Defense Nuclear Agency, Washington, D. C.

Schmidt, R. M., and K. A. Holsapple (1978b) A gravity-scaled energy parameter relating impact and explosive crater size (abstract). EOS (Trans. Am. Geophysics U.) 59, p. 1121.

Schmidt, R. M., and K. A. Holsapple (1979) Centrifuge crater scaling experiment II: Material strength effects, Report DNA 4999Z, Defense Nuclear Agency, Washington, D.C.

Schmidt, R. M., and K. A. Holsapple (1980a) Theory and experiments on centrifuge cratering, J. Geophys. Res., vol. 85, no. B1, pp. 235-252.

Schmidt, R. M., and K. A. Holsapple (1980b) Centrifuge crater scaling experiment III: H.O.B./D.O.B. effects, DNA Report, Contract DNA 001-78-C-0149, Defense Nuclear Agency, Washington, D.C.

Schmidt, R. M., H. E. Watson, and C. R. Wauchope (1979) Projectile density/target density correlation for impact cratering (abstract). EOS (Trans. Am. Geophysics U.) 60, p. 871.

Sedov, L. I. (1959) Similarity and Dimensional Methods in Mechanics, 4th edition, p. 256, Academic Press, New York.

Swift, R. P.(1977) Material strength degradation effect on cratering dynamics, in Impact and Explosion Cratering, edited by D. J. Roddy, R. O. Pepin, and R. B. Merrill, pp. 1025-1042, Pergamon Press, N.Y.

Truesdell, C., and W. Noll (1965) The non-linear field theories of mechanics, in Handbuch der Physik, vol. III/3, p. 36, edited by S. Flugge, Springer-Verlag, New York.

Vortman, L. J. (1968) Craters from surface explosions and scaling laws, J. Geophys. Res. vol. 73, pp. 4621-4636.

Vortman, L. J. (1977) Craters from surface explosions and energy dependence, a retrospective view , in Impact and Explosion Cratering, edited by D. J. Roddy, R. O. Pepin, and R. B. Merrill, pp. 1215-1229, Pergamon Press, New York.

White, J. W. (1971) Examination of cratering formulas and scaling methods, J. Geophys. Res., vol. 76, no. 35, pp. 8599-8603.

APPENDIX A

SDA Computer Printout

```
*******************************************************************

 -SDAPP
 ENTER+1=VOL,+2=RADIUS,+3=DEPTH
I  1
ALPHA=   .4720   BETA=   .1667
....ENTER +1 NO CHANGE,+2 CHANGE ALPHA BETA
I  2
ENTER ALPHA +0.1234
I 0.472
ENTER BETA +0.1234
I 0.0
ALPHA=   .4720   BETA=   .0000
          THERE ARE 226 DATA POINTS.
          ENTER DEPENDENT VARIABLE NAME
I PI1PI2BAR
ENTER 50 CHARACTER GRAPH HEADING
I ALL DATA. STRENGTH AND SIZE CORRECTIONS.
ENTER *RUN XY*
I RUN 76
ENTER PRINT CODE(0,1,2,3,4,5) NONE-MUCH
I 5
        5
```

```
STEPWISE LINEAR REGRESSION USING HOUSEHOLDER TRANSFORMATIONS - NWRGSN

         TOTAL NUMBER OF OBSERVATIONS =    226

         TOTAL NUMBER OF VARIABLES IS      5

         RESPONSE VARIABLE IS    5

         CONSTANT TERM (VAR. NO.  6) IS COMPUTED

         INDEPENDENT VARIABLES ALWAYS IN REGRESSION ARE     6

         SIGNIFICANCE LEVEL FOR ENTERING REGRESSION IS    .050

         SIGNIFICANCE LEVEL FOR LEAVING REGRESSION IS    .150
```

A

MEANS AND STANDARD DEVIATIONS

VAR. NO.	1	2	3	4	5
MEAN	1.4808E+00	1.6097E+01	2.1186E+02	3.1341E+03	-6.9339E-01
STD. DEV.	3.7288E+00	5.3619E+01	8.7175E+02	1.4913E+04	3.6862E-01

B

CORRELATION COEFFICIENTS

VAR. NO.	1	2	3	4	5
1	1.00000000	.94045675	.86765009	.80213460	.28303789
2	.94045675	1.00000000	.98058101	.94344764	-.03383713
3	.86765009	.98058101	1.00000000	.98954088	-.16269240
4	.80213460	.94344764	.98954088	1.00000000	-.22720944
5	.28303789	-.03383713	-.16269240	-.22720944	1.00000000

STEP NO. 1

INDEPENDENT VARIABLES ARE 6

STANDARD DEVIATION OF RESIDUALS = 3.69438471E-01

MULT. CORR. COEFF. SQUARED (R**2) = .00000000

C

REGRESSION COEFFICIENTS AND STANDARD DEVIATIONS

VAR. NO.	COEFFICIENT	STD. DEV.
6	-6.93389063594292E-01	2.45746815401868E-02

PARTIAL CORRELATION COEFFICIENTS

VAR. NO.	COEFFICIENT
1	.28303789
2	-.03383713
3	-.16269240
4	-.22720944

D

STEP NO. 2

INDEPENDENT VARIABLES ARE 6 1

VARIABLE ENTERING REGRESSION IS 1

STANDARD DEVIATION OF RESIDUALS = 3.55121700E-01

MULT. CORR. COEFF. SQUARED (R**2) = .08011045

E

ANOVA TABLE

SOURCE	SS	DF	MS	F
REGRESSION	2.4601179E+00	1	2.4601179E+00	1.9507494E+01
RESIDUALS	2.8248959E+01	224	1.2611142E-01	
CORRECTED TOTAL	3.0709076E+01	225		

REGRESSION COEFFICIENTS AND STANDARD DEVIATIONS

VAR. NO.	COEFFICIENT	STD. DEV.
6	-7.34822005724254E-01	2.54168573484879E-02
1	2.79805831888923E-02	6.33513708076897E-03

PARTIAL CORRELATION COEFFICIENTS

VAR. NO.	COEFFICIENT
2	-.92027379
3	-.05619034
4	-.29312702

F

THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COPY FURNISHED TO DDC

```
STEP NO.      3

          INDEPENDENT VARIABLES ARE     6     1     2

          VARIABLE ENTERING REGRESSION IS     2

          STANDARD DEVIATION OF RESIDUALS = 1.39261451E-01

          MULT. CORR. COEFF. SQUARED (R**2) =  .85916846

ANOVA TABLE
    SOURCE                 SS            DF        MS                F
  REGRESSION        2.6384270E+01     2   1.3192135E+01   6.8022604E+02
  RESIDUALS         4.3248066E+00   223   1.9393752E-02
  CORRECTED TOTAL   3.0709076E+01   225


REGRESSION COEFFICIENTS AND STANDARD DEVIATIONS
  VAR. NO.        COEFFICIENT                STD. DEV.
       6    -8.04957512983837E-01     1.01653170269991E-02
       1     2.69397480512508E-01     7.30872254277432E-03
       2    -1.78516078020389E-02     5.08264733486745E-04


          PARTIAL CORRELATION COEFFICIENTS
            VAR. NO.          COEFFICIENT
               3               .45200354
               4               .45052039
```

STEP NO. 4

G

INDEPENDENT VARIABLES ARE 6 1 2 3

VARIABLE ENTERING REGRESSION IS 3

STANDARD DEVIATION OF RESIDUALS = 1.24502930E-01

MULT. CORR. COEFF. SQUARED (R**2) = .88794135

ANOVA TABLE

SOURCE	SS	DF	MS	F
REGRESSION	2.7267859E+01	3	9.0892863E+00	5.8636851E+02
RESIDUALS	3.4412175E+00	222	1.5500980E-02	
CORRECTED TOTAL	3.0709076E+01	225		

REGRESSION COEFFICIENTS AND STANDARD DEVIATIONS

VAR. NO.	COEFFICIENT	STD. DEV.
6	-8.06538418016316E-01	9.09043844072976E-03
1	3.39608492144267E-01	1.13655632523870E-02
2	-3.25855931646049E-02	2.00373110084645E-03
3	6.36162271303732E-04	8.42601636377167E-05

PARTIAL CORRELATION COEFFICIENTS

VAR. NO.	COEFFICIENT
4	.02604549

CORRECTED REGRESSION COEFFICIENTS

VAR. NO.	COEFFICIENT	LAST CORRECTION
6	-8.06538418016334E-01	3.99834377524453E-14
1	3.39608492144277E-01	-6.20184007195945E-15
2	-3.25855931646053E-02	1.86671556796429E-14
3	6.36162271303743E-04	1.15617283220426E-14

DISPERSION MATRIX

H

VAR. NO.	6	1	2	3
6	5.3310E-03	-1.3046E-03	7.8695E-05	-1.1382E-06
1	-1.3046E-03	8.3334E-03	-1.3509E-03	5.0550E-05
2	7.8695E-05	-1.3509E-03	2.5901E-04	-1.0608E-05
3	-1.1382E-06	5.0550E-05	-1.0608E-05	4.5802E-07

RESIDUALS

OBS. NO.	OBSERVED	PREDICTED	RESIDUALS	STDZ RESIDUALS
1	-8.1570179E-01	-8.0653842E-01	-9.1633722E-03	-7.3599650E-02
2	-7.9544141E-01	-8.0653842E-01	1.1097009E-02	8.9130506E-02
3	-8.3271045E-01	-8.0653842E-01	-2.6172031E-02	-2.1021217E-01
4	-7.5062210E-01	-8.0653842E-01	5.5916315E-02	4.4911645E-01
5	-7.9263403E-01	-8.0653842E-01	1.3904390E-02	1.1167922E-01
6	-8.3412757E-01	-8.0653842E-01	-2.7589153E-02	-2.2159441E-01
7	-8.9249457E-01	-8.0653842E-01	-8.5956155E-02	-6.9039463E-01
8	-7.8156474E-01	-8.0653842E-01	2.4973682E-02	2.0058710E-01
9	-8.0029490E-01	-8.0653842E-01	6.2435146E-03	5.0147531E-02
10	-7.9505515E-01	-8.0653842E-01	1.1483266E-02	9.2232899E-02
11	-7.9270584E-01	-8.0653842E-01	1.3832573E-02	1.1110239E-01
12	-8.0397074E-01	-8.0653842E-01	2.5676782E-03	2.0623436E-02
13	-8.0583779E-01	-8.0653842E-01	7.0063147E-04	5.6274296E-03
14	-7.9717215E-01	-8.0653842E-01	9.3662641E-03	7.5229266E-02
15	-7.8830962E-01	-8.0653842E-01	1.8228794E-02	1.4641257E-01
16	-7.6540872E-01	-8.0653842E-01	4.1129694E-02	3.3035121E-01
17	-7.7264283E-01	-8.0653842E-01	3.3895589E-02	2.7224732E-01
18	-8.2023331E-01	-8.0653842E-01	-1.3694890E-02	-1.0999653E-01
19	-8.1598371E-01	-8.0653842E-01	-9.4452896E-03	-7.5863994E-02
20	-8.2561705E-01	-8.0653842E-01	-1.9078627E-02	-1.5323838E-01
21	-8.5755351E-01	-8.0653842E-01	-5.1015097E-02	-4.0975017E-01
22	-8.4284633E-01	-8.0653842E-01	-3.6307913E-02	-2.9162296E-01
23	-8.5773986E-01	-8.0653842E-01	-5.1201440E-02	-4.1124686E-01
24	-8.4246019E-01	-8.0653842E-01	-3.5921772E-02	-2.8852150E-01
25	-8.2185584E-01	-8.0653842E-01	-1.5317425E-02	-1.2302863E-01
26	-8.3235870E-01	-8.0653842E-01	-2.5820283E-02	-2.0738695E-01
27	-8.4036383E-01	-8.0653842E-01	-3.3825409E-02	-2.7168364E-01
28	-8.7943268E-01	-8.0653842E-01	-7.2894265E-02	-5.8548232E-01
29	-8.3100610E-01	-8.0653842E-01	-2.4467677E-02	-1.9652290E-01
30	-8.4980920E-01	-8.0653842E-01	-4.3270785E-02	-3.4754833E-01
31	-7.9750751E-01	-8.0653842E-01	9.0309086E-03	7.2535711E-02
32	-7.9893188E-01	-8.0653842E-01	7.6065365E-03	6.1095241E-02
33	-8.1607018E-01	-8.0653842E-01	-9.5317601E-03	-7.6558520E-02
34	-8.4002688E-01	-8.0653842E-01	-3.3488461E-02	-2.6897729E-01
35	-8.0470566E-01	-8.0653842E-01	1.8327586E-03	1.4720606E-02
36	-8.3954297E-01	-8.0653842E-01	-3.3004555E-02	-2.6509059E-01
37	-8.2310934E-01	-8.0653842E-01	-1.6570918E-02	-1.3309661E-01
38	-8.5210829E-01	-8.0653842E-01	-4.5569873E-02	-3.6601446E-01
39	-7.8250482E-01	-8.0653842E-01	2.4033602E-02	1.9303643E-01
40	-1.1487859E+00	-1.0295497E+00	-1.1923621E-01	-9.5769801E-01
41	-9.7873500E-01	-1.0295497E+00	5.0814728E-02	4.0814082E-01
42	-9.7617849E-01	-1.0295497E+00	5.3371235E-02	4.2867453E-01
43	-1.0447051E+00	-1.0295497E+00	-1.5155346E-02	-1.2172682E-01
44	-1.0299346E+00	-1.0295497E+00	-3.8487694E-04	-3.0913083E-03
45	-1.0064587E+00	-1.0295497E+00	2.3091072E-02	1.8546609E-01
46	-1.0087965E+00	-1.0295497E+00	2.0753195E-02	1.6668840E-01
47	-1.0289090E+00	-1.0295497E+00	6.4072760E-04	5.1462853E-03
48	-1.0752287E+00	-1.0295497E+00	-4.5679008E-02	-3.6689103E-01
49	-9.8442166E-01	-1.0295497E+00	4.5128065E-02	3.6246588E-01
50	-1.0283815E+00	-1.0295497E+00	1.1682767E-03	9.3835278E-03
51	-9.9703453E-01	-1.0295497E+00	3.2515194E-02	2.6116007E-01
52	-9.9115161E-01	-1.0295497E+00	3.8398122E-02	3.0841139E-01
53	-1.0107102E+00	-1.0295497E+00	1.8839486E-02	1.5131761E-01
54	-9.1209820E-01	-1.0295497E+00	1.1745153E-01	9.4336358E-01
55	-9.5846400E-01	-1.0295497E+00	7.1085732E-02	5.7095629E-01

56	-9.4137699E-01	-1.0295497E+00	8.8172733E-02	7.0819806E-01
57	-1.0289534E+00	-1.0295497E+00	5.9629068E-04	4.7893707E-03
58	-1.0203394E+00	-1.0295497E+00	9.2103131E-03	7.3976678E-02
59	-1.0394247E+00	-1.0295497E+00	-9.8749582E-03	-7.9315067E-02
60	-1.0719526E+00	-1.0295497E+00	-4.2402899E-02	-3.4057752E-01
61	-1.0425230E+00	-1.0295497E+00	-1.2973284E-02	-1.0420064E-01
62	-1.0254063E+00	-1.0295497E+00	4.1434289E-03	3.3279770E-02
63	-1.0528591E+00	-1.0295497E+00	-2.3309421E-02	-1.8721986E-01
64	-1.0198903E+00	-1.0295497E+00	9.6594559E-03	7.7584165E-02
65	-1.0330876E+00	-1.0295497E+00	-3.5379061E-03	-2.8416248E-02
66	-1.0369545E+00	-1.0295497E+00	-7.4048008E-03	-5.9474912E-02
67	-1.0200035E+00	-1.0295497E+00	9.5462077E-03	7.6674562E-02
68	-1.0119790E+00	-1.0137977E+00	1.8187352E-03	1.4607971E-02
69	-1.0262947E+00	-1.0148087E+00	-1.1485990E-02	-9.2254778E-02
70	-9.9909909E-01	-1.0148087E+00	1.5709580E-02	1.2617840E-01
71	-1.0400872E+00	-1.0139727E+00	-2.6114492E-02	-2.0975002E-01
72	-1.0212727E+00	-1.0148087E+00	-6.4640580E-03	-5.1918923E-02
73	-1.0225813E+00	-1.0295497E+00	6.9684157E-03	5.5969893E-02
74	-1.0200248E+00	-1.0295497E+00	9.5248823E-03	7.6503278E-02
75	-1.0665069E+00	-1.0295497E+00	-3.6957187E-02	-2.968[illegible]789E-01
76	-1.0819375E+00	-1.0295497E+00	-5.2387800E-02	-4.2077564E-01
77	-1.0212213E+00	-1.0295497E+00	8.3284121E-03	6.68[illegible]E-02
78	-1.0656744E+00	-1.0295497E+00	-3.6124645E-02	-2.9015[illegible]6E-01
79	-1.0186982E+00	-1.0112706E+00	-7.4275543E-03	-5.96[illegible]7E-02
80	-1.0078576E+00	-1.0112706E+00	3.4130159E-03	2.74[illegible]137E-02
81	-9.9926134E-01	-1.0295497E+00	3.0288388E-02	2.4327450E-01
82	-1.0137459E+00	-1.0295497E+00	1.5803872E-02	1.26[illegible]574E-01
83	-1.0727574E+00	-1.0295497E+00	-4.3207678E-02	-3.4704145E-01
84	-1.0208700E+00	-1.0295497E+00	8.6797184E-03	6.9714[illegible]E-02
85	-9.7338097E-01	-1.0295497E+00	5.6168760E-02	4.5114408E-01
86	-6.5897041E-01	-6.0853150E-01	-5.0438910E-02	-4.0512227E-01
87	-6.6645854E-01	-6.0853150E-01	-5.7927039E-02	-4.6526646E-01
88	-6.3497770E-01	-6.0853150E-01	-2.6446195E-02	-2.1241424E-01
89	-6.5779374E-01	-6.0853150E-01	-4.9262242E-02	-3.9567135E-01
90	-6.8904968E-01	-6.0853150E-01	-8.0518181E-02	-6.4671715E-01
91	-6.8770607E-01	-6.0853150E-01	-7.9174568E-02	-6.3592534E-01
92	-5.9070223E-01	-6.0853150E-01	1.7829275E-02	1.4320366E-01
93	-6.7136881E-01	-6.0853150E-01	-6.2837310E-02	-5.0470547E-01
94	-6.9526200E-01	-6.0853150E-01	-8.6730495E-02	-6.9661408E-01
95	-5.8663189E-01	-6.0853150E-01	2.1899608E-02	1.7589632E-01
96	-6.2622978E-01	-6.0853150E-01	-1.7698274E-02	-1.4215147E-01
97	-5.6100707E-01	-6.0853150E-01	4.7524432E-02	3.8171336E-01
98	-6.2612827E-01	-6.0853150E-01	-1.7596770E-02	-1.4133619E-01
99	-5.4423638E-01	-6.2104308E-01	7.6806700E-02	6.1690677E-01
100	-5.5274530E-01	-6.2023405E-01	6.7488751E-02	5.4206556E-01
101	-5.9840468E-01	-6.2023405E-01	2.1829365E-02	1.7533214E-01
102	-5.6913783E-01	-6.2023405E-01	5.1096222E-02	4.1040176E-01
103	-7.4875500E-01	-6.0853150E-01	-1.4022350E-01	-1.1262667E+00
104	-6.8348460E-01	-6.0853150E-01	-7.4953104E-02	-6.0201879E-01
105	-6.7056442E-01	-6.0853150E-01	-6.2032916E-02	-4.9824463E-01
106	-6.7853876E-01	-6.0853150E-01	-7.0007256E-02	-5.6229404E-01
107	-6.8832169E-01	-6.0853150E-01	-7.9790184E-02	-6.40869[illegible]E-01
108	-6.8772831E-01	-6.0853150E-01	-7.9196804E-02	-6.3610394E-01
109	-6.8325959E-01	-6.0853150E-01	-7.4728093E-02	-6.0021152E-01
110	-6.8161936E-01	-6.0853150E-01	-7.3087862E-02	-5.8703728E-01
111	-6.8218981E-01	-6.0853150E-01	-7.3658311E-02	-5.9161909E-01
112	-6.6542226E-01	-6.0853150E-01	-5.6890762E-02	-4.5694316E-01
113	5.4163431E-03	1.3421634E-01	-1.2879999E-01	-1.0345137E+00

114	-4.5227107E-01	-1.8301121E-01	-2.6925987E-01	-2.1626790E+00
115	-4.4505272E-01	-4.4070339E-01	-4.3493259E-03	-3.4933523E-02
116	-5.1199951E-01	-6.1090323E-01	9.8903724E-02	7.9438873E-01
117	-8.9556439E-01	-8.0653842E-01	-8.9025972E-02	-7.1505122E-01
118	-1.4379524E+00	-1.0265443E+00	-4.1140812E-01	-3.3044051E+00
119	6.3513996E-02	1.3421634E-01	-7.0702339E-02	-5.6787690E-01
120	-8.3428643E-02	-1.8301121E-01	9.9582564E-02	7.9984113E-01
121	-2.3430713E-02	1.1134950E-02	-3.4565662E-02	-2.7762931E-01
122	-5.2759760E-01	-6.1090323E-01	8.3315626E-02	6.6918606E-01
123	7.1213805E-02	1.3421634E-01	-6.3002530E-02	-5.0603251E-01
124	-2.9773320E-01	-4.4070339E-01	1.4297019E-01	1.1483279E+00
125	-1.6982998E-01	-1.8301121E-01	1.3181222E-02	1.0587078E-01
126	-6.6181003E-01	-5.8425196E-01	-7.7558074E-02	-6.2294175E-01
127	-5.3040084E-01	-5.8405030E-01	5.3649464E-02	4.3090925E-01
128	-1.8685484E-01	-1.8230521E-01	-4.5496294E-03	-3.6542348E-02
129	-1.9236134E+00	-1.0608545E+00	-8.6275888E-01	-6.9296271E+00
130	-3.6294610E-01	-3.9268001E-01	2.9733919E-02	2.3882104E-01
131	-4.0870359E-01	-4.9079647E-01	8.2092889E-02	6.5936512E-01
132	-4.4649215E-01	-5.2989437E-01	8.3402221E-02	6.6988159E-01
133	-5.0318906E-01	-5.9791361E-01	9.4724543E-02	7.6082180E-01
134	-5.4377100E-05	-1.8231171E-01	1.8225733E-01	1.4638799E+00
135	3.2547562E-01	1.9172926E-01	1.3374636E-01	1.0742427E+00
136	-4.4030628E-01	-2.7722562E-01	-1.6308066E-01	-1.3098540E+00
137	1.6674060E-01	1.3421634E-01	3.2524262E-02	2.6123290E-01
138	2.7940321E-01	1.9172926E-01	8.7673953E-02	7.0419188E-01
139	3.2466061E-02	3.9376883E-02	-6.9108217E-03	-5.5507302E-02
140	-6.3324210E-01	-2.7722562E-01	-3.5601648E-01	-2.8595028E+00
141	-5.9791414E-01	-7.0019488E-01	1.0228074E-01	8.2151272E-01
142	-1.5147924E+00	-1.6936370E+00	1.7884464E-01	1.4364693E+00
143	3.4019887E-01	3.9376883E-02	3.0082199E-01	2.4161840E+00
144	-1.2657914E+00	-7.0019488E-01	-5.6559653E-01	-4.5428371E+00
145	-1.7913103E+00	-1.7092118E+00	-8.2098480E-02	-6.5941002E-01
146	-1.7533011E+00	-1.3610706E+00	-3.9223046E-01	-3.1503713E+00
147	-7.7570518E-01	-1.2493291E+00	4.7362395E-01	3.8041189E+00
148	-9.2336873E-01	-7.9682085E-01	-1.2654788E-01	-1.0164249E+00
149	-5.3339765E-01	-6.9286695E-01	1.5946930E-01	1.2808477E+00
150	-1.5124108E-01	-3.3823728E-01	1.8699621E-01	1.5019422E+00
151	2.6061603E-02	-3.0130784E-01	3.2736944E-01	2.6294115E+00
152	2.1656237E-01	7.7298095E-03	2.0883256E-01	1.6773305E+00
153	-7.9932822E-01	-8.0653842E-01	7.2101935E-03	5.7911838E-02
154	-5.7964284E-01	-8.0653842E-01	2.2689558E-01	1.8224116E+00
155	-3.8427280E-01	-1.5869467E-01	-2.2557813E-01	-1.8118299E+00
156	-1.5200294E-01	-1.8230521E-01	3.0302270E-02	2.4338600E-01
157	8.5699411E-02	1.3419906E-01	-4.8499653E-02	-3.8954628E-01
158	9.9828506E-02	1.9231672E-01	-9.2438219E-02	-7.4285977E-01
159	-7.6921908E-01	-8.0653842E-01	3.7319343E-02	2.9974670E-01
160	4.9036771E-02	1.3137122E-01	-8.2334445E-02	-6.6130528E-01
161	-5.1884117E-01	-5.8653770E-01	6.7696531E-02	5.4373444E-01
162	-4.5530524E-01	-5.3478686E-01	7.9481616E-02	6.3839153E-01
163	-5.8180129E-01	-5.8653770E-01	4.7364155E-03	3.8042603E-02
164	-5.2142628E-01	-5.8653770E-01	6.5111423E-02	5.2297100E-01
165	-5.9726827E-01	-5.8653770E-01	-1.0730564E-02	-8.6187244E-02
166	-1.6282667E-01	-9.1847807E-02	-7.0978866E-02	-5.7009795E-01
167	-9.2337837E-02	7.3043017E-03	-9.9642139E-02	-8.0031963E-01
168	-4.9130781E-02	6.7186294E-02	-1.1631707E-01	-9.3425170E-01
169	-6.6354480E-02	1.1577091E-01	-1.8212539E-01	-1.4628201E+00
170	-6.7084731E-02	9.2099853E-02	-1.5918458E-01	-1.2785609E+00
171	-2.0242564E-01	-1.0889353E-01	-9.3532106E-02	-7.5124421E-01

172	-4.4779169E-02	1.4113314E-01	-1.8591231E-01	-1.4932364E+00
173	-5.9433550E-01	-5.8653770E-01	-7.7977993E-03	-6.2631452E-02
174	-3.2993440E-01	-3.1724073E-01	-1.2693676E-02	-1.0195484E-01
175	-1.0119195E-01	-1.0889353E-01	7.7015840E-03	6.1858656E-02
176	2.0120376E-01	1.4113314E-01	6.0070621E-02	4.8248359E-01
177	3.1744857E-01	1.9590551E-01	1.2154305E-01	9.7622644E-01
178	4.0863477E-01	8.7786825E-02	3.2084794E-01	2.5770313E+00
179	-3.9545566E-01	-5.8957609E-01	1.9412043E-01	1.5591636E+00
180	-4.9148735E-01	-5.3478686E-01	4.3299512E-02	3.4777906E-01
181	-7.7032515E-01	-8.0653842E-01	3.6213265E-02	2.9086275E-01
182	-7.3235782E-01	-8.0653842E-01	7.4180601E-02	5.9581409E-01
183	-8.2440093E-01	-8.0653842E-01	-1.7862509E-02	-1.4347059E-01
184	-7.8890395E-01	-8.0653842E-01	1.7634464E-02	1.4163894E-01
185	-8.2433503E-01	-8.0653842E-01	-1.7796612E-02	-1.4294131E-01
186	-7.7640083E-01	-8.0653842E-01	3.0137583E-02	2.4206324E-01
187	-7.2337132E-01	-8.0653842E-01	8.3167094E-02	6.6799306E-01
188	-6.7475131E-01	-8.0653842E-01	1.3178711E-01	1.0585061E+00
189	-7.8938389E-01	-8.0653842E-01	1.7154528E-02	1.3778413E-01
190	-7.8877802E-01	-8.0653842E-01	1.7760397E-02	1.4265043E-01
191	-7.7512097E-01	-8.0653842E-01	3.1417443E-02	2.5234300E-01
192	-7.7324615E-01	-8.0653842E-01	3.3292263E-02	2.6740144E-01
193	-7.1977959E-01	-8.0653842E-01	8.6758826E-02	6.9684164E-01
194	-6.9999581E-01	-8.0653842E-01	1.0654261E-01	8.5574377E-01
195	-8.8314655E-01	-8.0653842E-01	-7.6608134E-02	-6.1531190E-01
196	-8.7146796E-01	-8.0653842E-01	-6.4929540E-02	-5.2151013E-01
197	-3.4307318E-01	-5.1868717E-01	1.7561398E-01	1.4105209E+00
198	-5.1841154E-01	-6.7651580E-01	1.5810426E-01	1.2698838E+00
199	-7.2291387E-01	-8.0653842E-01	8.3624547E-02	6.7166730E-01
200	-7.8657006E-01	-8.0653842E-01	1.9968354E-02	1.6038461E-01
201	-8.0576483E-01	-8.0653842E-01	7.7358921E-04	6.2134217E-03
202	-7.6442798E-01	-8.0653842E-01	4.2110440E-02	3.3822851E-01
203	-8.9170062E-01	-8.0653842E-01	-8.5162206E-02	-6.8401769E-01
204	-7.9321335E-01	-8.0653842E-01	1.3325065E-02	1.0702612E-01
205	-7.7267172E-01	-8.0653842E-01	3.3866702E-02	2.7201530E-01
206	-7.6632638E-01	-8.0653842E-01	4.0212041E-02	3.2298068E-01
207	-7.1210333E-01	-8.0653842E-01	9.4435085E-02	7.5849689E-01
208	-8.3929472E-01	-8.0653842E-01	-3.2756301E-02	-2.6309663E-01
209	-9.8496764E-01	-8.0653842E-01	-1.7842922E-01	-1.4331327E+00
210	-4.9830674E-01	-8.0653842E-01	3.0823168E-01	2.4756982E+00
211	-7.8216363E-01	-8.0653842E-01	2.4374788E-02	1.9577682E-01
212	-7.5453134E-01	-8.0653842E-01	5.2007076E-02	4.1771769E-01
213	-7.6659585E-01	-8.0653842E-01	3.9942565E-02	3.2081626E-01
214	-8.0424844E-01	-8.0653842E-01	2.2899738E-03	1.8392931E-02
215	-7.6701875E-01	-8.0653842E-01	3.9519669E-02	3.1741959E-01
216	-8.0509538E-01	-8.0653842E-01	1.4430400E-03	1.1590410E-02
217	-7.8625301E-01	-8.0653842E-01	2.0285404E-02	1.6293114E-01
218	-8.1494964E-01	-8.0653842E-01	-8.4112214E-03	-6.7558421E-02
219	-8.3195535E-01	-8.0653842E-01	-2.5416934E-02	-2.0414728E-01
220	-6.5858899E-01	-8.0653842E-01	1.4794943E-01	1.1883208E+00
221	-5.7356310E-01	-8.0653842E-01	2.3297532E-01	1.8712437E+00
222	-7.7720185E-01	-8.0653842E-01	2.9336570E-02	2.3562956E-01
223	-6.0459205E-01	-8.0653842E-01	2.0194637E-01	1.6220210E+00
224	-7.9150430E-01	-8.0653842E-01	1.5034122E-02	1.2075316E-01
225	-8.1954868E-01	-8.0653842E-01	-1.3010261E-02	-1.0449763E-01
226	-8.6460175E-01	-8.0653842E-01	-5.8063334E-02	-4.6636118E-01

NWRGSN SUCCESSFUL COMPLETION CODE = 0
I>

```
C REPLACE,PROGRAM=NONLN5
C -GO
1ENTRY TO NW021

 ITERATIONS       0    CALLS TO RESID        1

 RESIDUAL SUM-OF-SQUARES      2.3007316118009E+01
 X =
    5.0000000000000E-01    1.6700000000000E-01   -2.2887080800001E+00    7.091117
01E+00   -5.5409696099999E+00
    .0                     .0                     1.7957437200000E-01    .0
         .0
    .0                     .0
 RESIDUALS
    1.8780986672443E-01    2.5143556293899E-01    2.2365599868852E-01    3.750979
37E-02    1.2323433399816E-01
    1.7644301280151E-02    6.0823583253252E-02   -6.2015355960862E-01   -9.255311
57E-01   -3.2242604222437E-01
   -9.8217509204971E-02   -5.5418112168119E-01   -1.3313209942502E+00   -4.863787
18E-01   -7.6290111966692E-02
   -3.9271767599934E-01   -1.3411043044615E-01   -4.6864930328052E-01    1.678968
18E-02   -2.7523655515084E-01
   -2.7488794584795E-01    2.0771269406623E-01   -1.6712872837139E-01   -2.008748
96E+00   -5.9090900871247E-02
    7.7298545922111E-02    8.6749836460015E-02   -1.4258104180394E-01    1.008712
33E-01   -5.1945916278004E-03
   -4.2047483801442E-01   -2.4869073541666E-01   -1.1128023304164E-01   -2.366601
60E-01   -8.6472597147252E-01
    3.7230091716425E-01    1.8230499822792E-01    4.7192082093663E-01   -1.165543
84E+00   -4.4191959130137E-01
   -8.0620744097166E-01    1.2387029819866E+00   -1.2887808224499E-01    5.017288
94E-01    4.1689514592232E-01
    7.2132254060427E-01    2.7828358255730E-01   -3.3258916116808E-01    1.732551
43E-01    1.6562696916426E+00
    4.2390320162581E-03   -3.0158962987120E-02    6.3534370228429E-01    9.888713
20E-02   -3.0724687599802E-01
    1.8774001172070E-01    2.1497864179805E-01    2.5187810707093E-01   -4.480960
81E-02   -7.8559755446278E-02
   -1.4053923521437E-01   -2.3291603795563E-01   -1.0546845355789E-01    6.911246
30E-02    6.5785753010683E-03
    2.7983616474105E-02   -2.1674757739607E-01   -1.6139490356139E-01   -2.470093
29E-01   -9.7354124152563E-01
   -3.7298268269306E-01    4.9956048922055E-02   -1.7438113070435E-01   -1.593751
 7E-01   -4.9780492679541E-01
    2.6314308524391E-01    3.8660685781960E-01    4.0398771027062E-01    3.770374
  -01    6.4177765570637E-01
    4.5728140654082E-01    1.3209588852091E-01    2.5735900322584E-01    3.433443
69E-01    2.4656872784807E-01
    3.3914808232748E-01    2.1782826184339E-01    9.2668251279808E-01    4.743485
97E-01    1.6327580930833E-01
    2.4501060962315E-01    1.4551072099574E-01    2.5588346122260E-01    1.655349
35E-01    2.7748683942179E-01
    2.6849857799025E-01    2.7281551156188E-01
```

THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COPY FURNISHED TO DDC

```
 ITERATIONS      3    CALLS TO RESID      7

 RESIDUAL SUM-OF-SQUARES     2.29838577950666E+01
 X =
   4.99406394903747E-01    1.66728542985390E-01   -2.29400239060019E+00    7.092188
07E+00  -5.53906566686916E+00
    .0                      .0                     1.79843867319264E-01     .0
            .0
    .0                      .0
 RESIDUALS
   2.03702750676911E-01    2.67328446891469E-01    2.39548882641003E-01    5.316260
80E-02    1.38887149070200E-01
   3.30653339991329E-02    7.62459044508716E-02   -6.10859981739221E-01   -9.164842
30E-01   -3.11814504761294E-01
  -8.63468715912532E-02   -5.40719617484541E-01   -1.31595018345502E+00   -4.770851
18E-01   -6.71931797686653E-02
  -3.84079352326737E-01   -1.22239792832438E-01   -4.59355726411129E-01    2.740121
64E-02   -2.66139622952821E-01
  -2.63785711956899E-01    2.18148305463025E-01   -1.58495582639901E-01   -1.993330
9E+00   -4.93836478504832E-02
   8.76842572492923E-02    9.74279184386049E-02   -1.13999395258212E-01    1.100017
34E-01    8.41594487899133E-03
  -3.91770674020052E-01   -2.39397158547259E-01   -9.76696965348500E-02   -2.158404
59E-01   -8.36021807478147E-01
   4.05616818695194E-01    1.90804878390878E-01    4.92740518514236E-01   -1.132227
90E+00   -4.34448447979776E-01
  -7.81690568828850E-01    1.26650548632139E+00   -9.54680576062970E-02    5.350218
53E-01    4.46659945871076E-01
   7.50461049144128E-01    3.00118226419347E-01   -3.19127656971432E-01    1.867166
99E-01    1.68526262070432E+00
   1.24289393984407E-02   -1.94493237641562E-02    6.52192466408607E-01    1.117605
33E-01   -2.97500330510042E-01
   2.04479696016296E-01    2.29098985344839E-01    2.64537006213954E-01   -1.888069
13E-02   -5.56614051616293E-02
  -1.19817458449340E-01   -2.00259596791440E-01   -7.65256470430842E-02    8.786879
65E-02    2.22745827773432E-02
   4.19196197445513E-02   -1.88449321702279E-01   -1.36035956300829E-01   -2.239117
99E-01   -9.41843425628363E-01
  -3.42406150869792E-01    7.00667800352655E-02   -1.57530455106707E-01   -1.444764
47E-01   -4.68384379049581E-01
   2.74695605073302E-01    3.97545748834830E-01    4.15029701212156E-01    3.883719
15E-01    6.52322612939621E-01
   4.65343066070330E-01    1.40448170144653E-01    2.66097420318843E-01    3.531374
01E-01    2.55660214742335E-01
   3.47272253184791E-01    2.27876641962392E-01    9.39937010630934E-01    4.865980
50E-01    1.75300826322825E-01
   2.57035626637645E-01    1.57305244034745E-01    2.67677944261610E-01    1.775597
11E-01    2.89511611332273E-01
   2.83024635696989E-01    2.87341569268619E-01

 IF =      0
   2.29838578E+01
   4.99406395E-01     1.66728543E-01    -2.29400239E+00
   7.09218834E+00    -5.53906567E+00     .0
    .0                1.79843867E-01     .0
    .0                 .0                .0
STOP
G-BYE
```

2. $1^{st}, 2^{nd}$, AND 5^{th} DEGREE TERMS ONLY.

APPENDIX B

SUMMARY OF CRATER RUPTURE DATA COMPENDIUM FOR SDA

ID NUMBER	NUMBER OF SHOTS	NAME	DATE	SIZE	TYPE
101-124	24	Russian Data	1955-1960	3.06-9.18 GM	TNT
201-230	30	Gravity Effects on Crater Formation	1965	6 GR	SQUIB(S-68)
301	1	Johnie Boy	1962	.5 KT	Nuclear
302-311	10	Cratering in Pacific Atoll	1952-1963	.0013-150 KT	Nuclear
401-434	34	Centrifuge Cratering	1976-1977	.22-.65 GM	E106
				.46 GM	E1A(6)
				.64 GM	E1A(8)
				1.7 GM	PBN6
				.4-4 GM	PETN
501	1	Danny Boy	1962	.42 KT	Nuclear
502	1	Mine Shaft	1969	100 ST	TNT
503	1	Mine Ore	1968	100 ST	TNT
504	1	Mine Under	1968	100 ST	TNT
505	1	Burst Charge	1963	20 ST	TNT
506	1	Mine Throw I	1971	118 ST	ANFO
507-509	3	Surface Charge Explos.	1959-1961	5-100 ST	TNT
510-587	78	Panama Canal Series	1946-1947	8-200 LB	TNT
588-591	4	Pre-Gondola I	1966	18.2 ST	NM
592-594	3	Diamond Ore	1973	10 ST	NM
595-617	23	Sandia I, II	1958-1959	256 LB	TNT
618-635	18	Little Ditch	1957-1958	2 LB	TNT
636-648	13	Single Charge	1966-1969	64 LB	TNT
649	1	Surface Burst	1961	100 ST	TNT
650-656	7	Pre-Buggy I	1962-1963	1000 LB	NM
657-680	24	Project Mole	1952-1954	256 LB	TNT
681-702	22	Project Toboggan	1961	8 LB	TNT
703	1	Shooter	1960	500 ST	TNT
704-706	3	Project Stagecoach	1960	20 ST	TNT
707-710	4	Calibration Series	1966	909 LB	NM
711	1	Pre-Schooner II	1965	85.5 ST	NM
712-720	9	Pre-Mine Throw IV	1973-1974	256-1000 LB	TNT
				.5-102 ST	NM
721-730	10	Operation Buckboard	1964	1000 LB	TNT
731-739	9	DRES (SES Canada)	1959-1963	.25-500 ST	TNT
740-802	63	Bureau of Mines	1956	.9-20.8 LB	Dynamite
803	1	Sedan	1962	100 KT	Nuclear
804	1	Schooner	1968	35 KT	Nuclear
805	1	Palanquin	1965	4.3 KT	Nuclear
806	1	Cabriolet	1968	2.3 KT	Nuclear
807-809	3	Flat Top I, II, III	1964	20 ST	TNT
810	1	Jangle S	1951	1.2 KT	Nuclear
811	1	Jangle U	1951	1.2 KT	Nuclear
812	1	Teapot Ess	1955	1.2 KT	Nuclear
813	1	Neptune	1954	115 KT	Nuclear

ID NUMBER	NUMBER OF SHOTS	NAME	DATE	SIZE	TYPE
814-816	3	Distant Plain	1967	20-100 ST	TNT
817-826	11	Mine Shaft Calibration Series	1968	.5 ST	TNT
827-830	4	Pre-Schooner	1964	20 ST	NM
831-833	3	Operation Buckboard	1960	20 ST	TNT
834	1	Snow Ball	1964	500 ST	TNT
835	1	Prairie Flat	1968	500 ST	TNT
836-857	22	Cratering in Loess and Clay	1956-1957	.5-1 LB	C-4
858	1	Dial Pack	1970	500 ST	TNT
859-881	23	Air Vent Series	1963-1964	.032-20 ST	TNT
882-893	12	Operation Jangle	1951	.108-20 ST	TNT
				176 LB	Pentolite
894-902	9	Multiple Cratering	1965	4000 LB	TNT
903-968	66	Underground Explosions	1949-1951	8-2560 LB	TNT
969-982	14	Soil Rock Interface	1957-1958	20 ST	C-4
				54 LB	Dynamite
				256 LB	TNT
983-986	4	Stemming Effects - HE Charges	1957	20 ST	C-4
987-1008	21	Stemming Underground Explosions	1957	20 ST	C-4
1009-1049	41	Partially Conf. Expl.	1955	20 ST	C-4
				54 LB	Dynamite
1050	1	Russian Event	1961	110000 ST	Nuclear
1051-1069	19	Ammonium Nitrate Cratering		50 LB	AN
				50 LB	TNT
1070-1081	12	Cratering in Sand	1962	4 LB	TNT
1082-1102	21	WES Stem. Series	1957-1960	10 LB	TNT
1103-1225	123	Colorado School of Mines (Underground Explosions)	1948-1949	.75-239 LB	C-4
				1.12-1080 LB	TNT
1226-1262	37	Spherical Charges	1953	.96-5.27 LB	Pentolite
1263-1298	35	Fort Churchill Tests	1956	.75-14.89 LB	C-4, C-3
1299-1379	81	Fort Churchill Charges	1957	2.65 LB	Pentolite
				1.98 LB	C-4, C-3
1380-1386	7	Project Trinidad	1973	2000 LB	AN
1387-1397	11	Shell Explosions	1971	4-16 LB	TNT
1398-1412	15	Railroad Voln. Program	1958	54-540 LB	TNT
1413-1431	19	Project Zulu	1968	.75 LB	C-4, C-3
1432-1436	6	Middle Gust I, II, III, IV, V	1971-1972	20-100 ST	TNT
1437-1439	3	Mixed Co. I, II, III	1972	20-500 ST	TNT
1440-1458	19	Middle Gust-Mixed Co.	1971-1972	1000 LB	TNT
1459-1463	5	Project ESSEX I	1973	1000 LB	TNT
1464-1469	6	Cratering in Playa	1970-1971	8,1000 LB	TNT
1470-1479	10	Cratering in Desert Alluvium	1958-1959	256 LB	TNT
9001-9404	404	A. J. Piekutowski-Sand	1977	1.7 GM	PBN6
				.4-.5 GM	PETN

DEPARTMENT OF THE NAVY (Continued)

Naval Surface Weapons Center
ATTN: Tech Library & Info Services Branch

Naval War College
ATTN: Code E-11

Naval Weapons Evaluation Facility
ATTN: Code 10

Office of Naval Research
ATTN: Code 715
ATTN: Code 474, N. Perrone

Office of the Chief of Naval Operations
ATTN: OP 981
ATTN: OP 03EG

Strategic Systems Project Office
Department of the Navy
ATTN: NSP-43

DEPARTMENT OF THE AIR FORCE

Air Force Institute of Technology
Air University
ATTN: Library

Air Force Systems Command
ATTN: DLW

Air Force Weapons Laboratory
Air Force Systems Command
ATTN: NTE, M. Plamondon
ATTN: DYT
ATTN: NTES-C, R. Henny
ATTN: SUL

Assistant Chief of Staff
Intelligence
Department of the Air Force
ATTN: INT

Ballistic Missile Office/MN
Air Force Systems Command
ATTN: MMH

Deputy Chief of Staff
Research, Development & Acq
Department of the Air Force
ATTN: AFRDQSM

Deputy Chief of Staff
Logistics & Engineering
Department of the Air Force
ATTN: LEEE

Foreign Technology Division
Air Force Systems Command
ATTN: NIIS Library

Rome Air Development Center
Air Force Systems Command
ATTN: TSLD

DEPARTMENT OF THE AIR FORCE (Continued)

Strategic Air Command/XPFS
Department of the Air Force
ATTN: NRI-STINFO Library

DEPARTMENT OF ENERGY

Department of Energy
Albuquerque Operations Office
ATTN: CTID

Department of Energy
Nevada Operations Office
ATTN: Mail & Records for Technical Library

OTHER GOVERNMENT

Central Intelligence Agency
ATTN: OSI/NED, J. Ingley

Department of the Interior
Bureau of Mines
ATTN: Tech Lib

Federal Emergency Management Agency
ATTN: Hazard Eval & Vul Red Div

DEPARTMENT OF ENERGY CONTRACTORS

Lawrence Livermore National Laboratory
ATTN: W. Crowley
ATTN: L-10, H. Kruger (Class L-94)
ATTN: J. Nutt
ATTN: Technical Information Dept Library

Los Alamos National Scientific Laboratory
ATTN: MS362, Librarian
ATTN: M. Henderson
ATTN: G. Spillman
ATTN: MS 364
ATTN: R. Bridwell
ATTN: MS 670, J. Hopkins

Oak Ridge National Laboratory
ATTN: Central Research Library
ATTN: Civil Def Res Proj

Sandia National Laboratories
Livermore Laboratory
ATTN: Library & Security Classification Div

Sandia National Laboratories
ATTN: A. Chabai
ATTN: 3141

DEPARTMENT OF DEFENSE CONTRACTORS

Acurex Corp
ATTN: J. Stockton

Aerospace Corp
ATTN: Technical Information Services

Agbabian Associates
ATTN: M. Agbabian

DISTRIBUTION LIST

DEPARTMENT OF DEFENSE

Assistant to the Secretary of Defense
Atomic Energy
ATTN: Executive Assistant

Defense Advanced Rsch Proj Agency
ATTN: TIO

Defense Intelligence Agency
ATTN: DB-4C, E. O'Farrell
ATTN: DT-1C
ATTN: DB-4N

Defense Nuclear Agency
2 cy ATTN: SPSS
4 cy ATTN: TITL

Defense Technical Information Center
12 cy ATTN: DD

Field Command
Defense Nuclear Agency
ATTN: FCTK
ATTN: FCPR

Field Command
Defense Nuclear Agency
Livermore Division
ATTN: FCPRL

Interservice Nuclear Weapons School
ATTN: TTV

Joint Strat Tgt Planning Staff
ATTN: NRI-STINFO Library
ATTN: JLA

NATO School (SHAPE)
ATTN: U.S. Documents Officer

Under Secy of Def for Rsch & Engrg
Department of Defense
ATTN: Strategic & Space Systems (OS)

DEPARTMENT OF THE ARMY

BMD Advanced Technology Center
Department of the Army
ATTN: 1CRDABH-X
ATTN: ATC-T

Chief of Engineers
Department of the Army
ATTN: DAEN-RDM
ATTN: DAEN-MCE-D

Harry Diamond Laboratories
Department of the Army
ATTN: DELHD-N-P
ATTN: DELHD-I-TL

U.S. Army Ballistic Research Labs
ATTN: DRDAR-BLT, W. Taylor
ATTN: DRDAR-TSB-S
ATTN: DRDAR-BLE, J. Keefer
ATTN: DRDAR-BLV

DEPARTMENT OF THE ARMY (Continued)

U.S. Army Concepts Analysis Agency
ATTN: MOCA-ADL

U.S. Army Engineer Center
ATTN: DT-LRC

U.S. Army Engineer Div Huntsville
ATTN: HNDED-SR

U.S. Army Engineer Div Ohio River
ATTN: ORDAS-L

U.S. Army Engr Waterways Exper Station
ATTN: WESSA, W. Flathau
ATTN: Library
ATTN: WESSD, G. Jackson
ATTN: J. Strange
ATTN: WESSE, L. Ingram

U.S. Army Material & Mechanics Rsch Ctr
ATTN: Technical Library

U.S. Army Materiel Dev & Readiness Cmd
ATTN: DRXAM-TL

U.S. Army Missile R&D Command
ATTN: RSIC

U.S. Army Nuclear & Chemical Agency
ATTN: Library

DEPARTMENT OF THE NAVY

David Taylor Naval Ship R & D Ctr
ATTN: Code L42-3

Naval Construction Battalion Center
Civil Engineering Laboratory
ATTN: Code L08A
ATTN: Code L51, S. Takahashi

Naval Electronic Systems Command
ATTN: PME 117-21

Naval Facilities Engineering Command
ATTN: Code 04B
ATTN: Code 09M22C
ATTN: Code 03T

Naval Material Command
ATTN: MAT 08T-22

Naval Postgraduate School
ATTN: Code 0142, Library

Naval Research Laboratory
ATTN: Code 2627

Naval Sea Systems Command
ATTN: SEA-09G53

Naval Surface Weapons Center
White Oak Laboratory
ATTN: Code F31

DEPARTMENT OF DEFENSE CONTRACTORS (Continued)

Applied Theory, Inc
2 cy ATTN: J. Trulio

AVCO Research & Systems Group
ATTN: Library A830

BDM Corp
ATTN: T. Neighbors
ATTN: Corporate Library

Boeing Co
ATTN: R. Schmidt
ATTN: Aerospace Library

California Research & Technology, Inc
ATTN: Library
ATTN: K. Kreyenhagen
ATTN: S. Schuster

California Research & Technology, Inc
ATTN: D. Orphal

Calspan Corp
ATTN: Library

Civil Systems, Inc
ATTN: J. Bratton

University of Denver
Colorado Seminary
Denver Research Institute
ATTN: J. Wisotski

EG&G Washington Analytical Services Center, Inc
ATTN: Library

Eric H. Wang Civil Engineering Rsch Fac
University of New Mexico
ATTN: N. Baum

Gard, Inc
ATTN: G. Neidhardt

General Electric Company-TEMPO
ATTN: DASIAC

IIT Research Institute
ATTN: Documents Library

Institute for Defense Analyses
ATTN: Classified Library

Kaman Avidyne
ATTN: Library

Kaman Sciences Corp
ATTN: Library

Lockheed Missiles & Space Company, Inc
ATTN: Technical Information Center
ATTN: T. Geers

Lockheed Missiles & Space Company, Inc
ATTN: TIC-Library

DEPARTMENT OF DEFENSE CONTRACTORS (Continued)

McDonnell Douglas Corp
ATTN: R. Halprin

Merritt Cases, Inc
ATTN: J. Merritt
ATTN: Library

Nathan M. Newmark Consult Eng Svcs
University of Illinois
ATTN: N. Newmark

Pacific-Sierra Research Corp
ATTN: H. Brode

Pacifica Technology
ATTN: R. Allen

Physics International Co
ATTN: J. Thomsen
ATTN: Technical Library
ATTN: L. Behrmann
ATTN: E. Moore
ATTN: F. Sauer

R & D Associates
ATTN: Technical Information Center
ATTN: W. Wright, Jr.
ATTN: J. Carpenter
ATTN: A. Latter
ATTN: R. Port
ATTN: J. Lewis
ATTN: C. MacDonald
ATTN: P. Haas

Science Applications, Inc
ATTN: H. Wilson
ATTN: Technical Library

Science Applications, Inc
ATTN: D. Maxwell
ATTN: D. Bernstein

Southwest Research Institute
ATTN: W. Baker
ATTN: A. Wenzel

SRI International
ATTN: G. Abrahamson
ATTN: B. Gasten

Systems, Science & Software, Inc
ATTN: Library
ATTN: D. Grine
ATTN: K. Pyatt
ATTN: T. Cherry
ATTN: R. Lafrenz
ATTN: T. Riney

Terra Tek, Inc
ATTN: S. Green
ATTN: Library

Tetra Tech, Inc
ATTN: L. Hwang
ATTN: Library

DEPARTMENT OF DEFENSE CONTRACTORS (Continued)

TRW Defense & Space Sys Group
2 cy ATTN: N. Lipner
ATTN: I. Alber
ATTN: Technical Information Center
ATTN: D. Baer
ATTN: R. Plebuch

TRW Defense & Space Sys Group
ATTN: E. Wong
ATTN: P. Dai

Universal Analytics, Inc
ATTN: E. Field

DEPARTMENT OF DEFENSE CONTRACTORS (Continued)

Weidlinger Associates, Consulting Engineers
ATTN: M. Baron
ATTN: J. Wright
ATTN: I. Sandler

Weidlinger Associates, Consulting Engineers
ATTN: J. Isenberg

Westinghouse Electric Corp
ATTN: W. Volz

END